Electeng 311

Electronics Systems Design

Magnetics

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NEW ZEALAND

Contents

- What is magnetics?
- Magnetics revision
- Magnetics – Laws
- Magnetic circuits and core saturation
- Ideal transformer
- Coupling factor
- Practical transformer

Learning outcomes

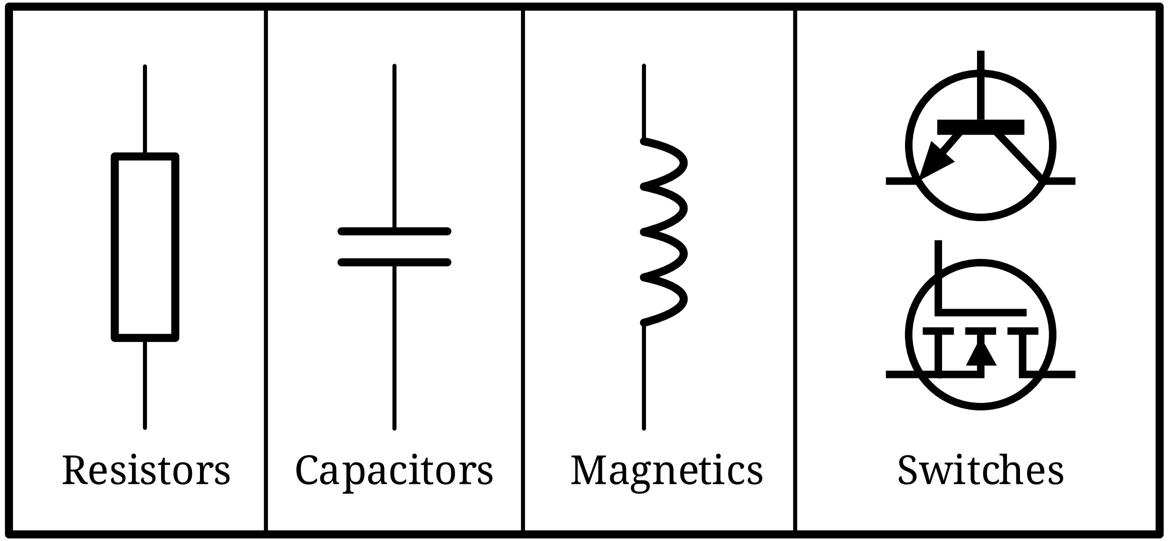
- Revise magnetic concepts and laws.
- Understand core saturation and the effect of air gaps.
- Understand and derive ideal transformer models.
- Understand the constraints in practical transformer model.

Magnetics

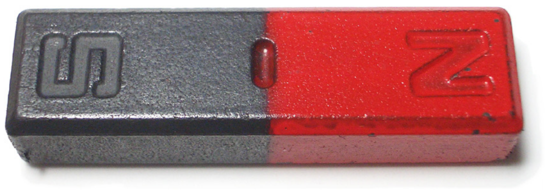
Magnetic circuits

Magnetics – introduction

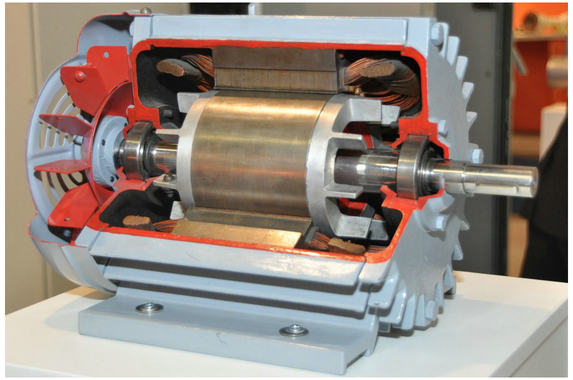
- Everything in electrical engineering are a combination of the basic elements.
- Magnetics encompass a wide range of devices that create or manipulate magnetic fields.



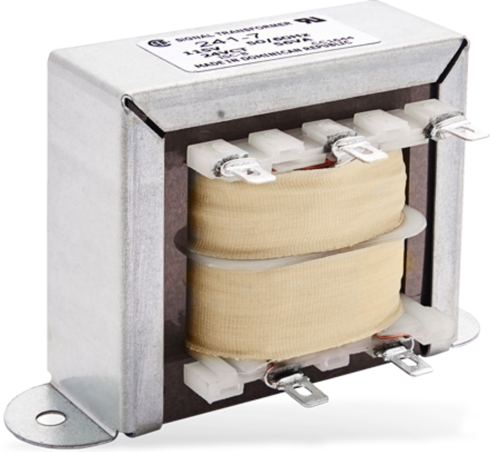
- Passive and active elements in electrical circuits



- A bar magnet [1]



- An electric motor [2]

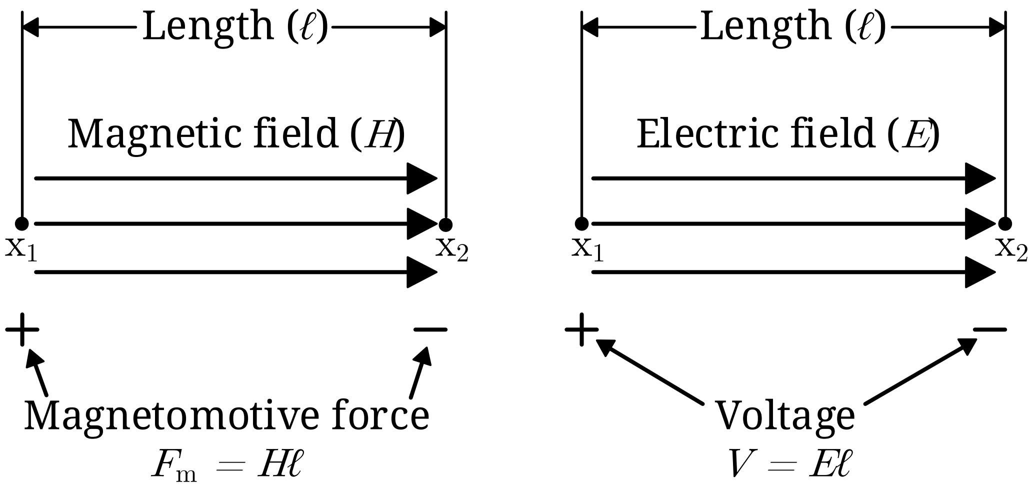


- Transformers [3] and [4]

[1] https://commons.wikimedia.org/wiki/File:Bar_magnet.jpg
[2] https://www.hisour.com/electric-motor-40853/
[3] https://nz.mouser.com/new/bel-signal-transformer/signal-transformer-two-4-one-power-transformers/
[4] https://the-rsgroup.com/the-1000th-power-transformer/

Magnetics – magnetomotive force

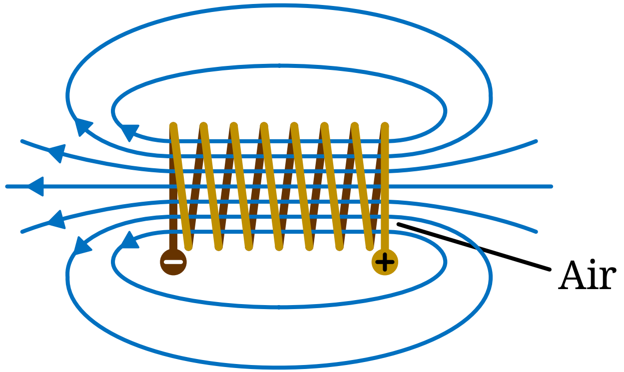
- Magnetic fields are analogous to electric fields.
- An electric field is formed when there is a difference in electric potential between two points. The electric potential difference is called electromotive force or voltage (V) .
- **Magnetomotive force** (F_m) can be considered as 'magnetic potential difference' between two points.
- A **magnetic field strength** (H) is generated when F_m is present between two points separated by a distance (ℓ) as given by:
- $F_\mathrm{m} = \int_{x_1}^{x_2} H \cdot d\ell$
- If H is assumed to be uniform, the equation is reduced to:
- $F_\mathrm{m} = H\ell$
- Similarly, the voltage in an uniform electric field (E) is given by:
- $V = E\ell$



- Magnetomotive force and voltage

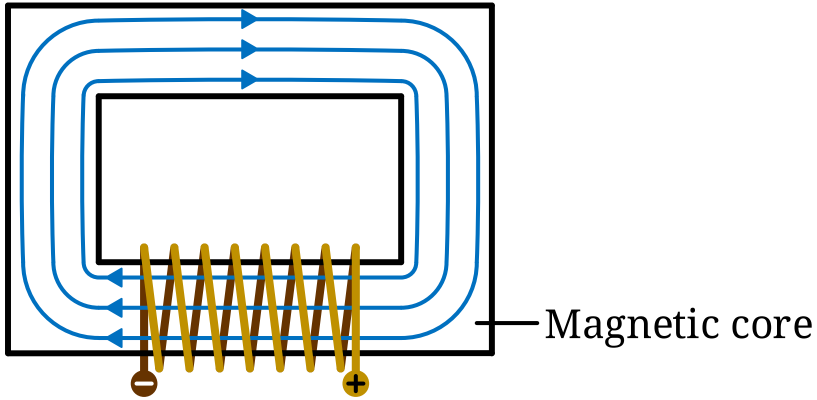
Magnetics – magnetic field

- Note that B and H are referring to two different things.
- Given the same coil wound in air and a magnetic material:
- **Magnetic field strength** (H) is dependent only on the magnetomotive force applied to the winding.
- **Magnetic flux density** (B) is dependent on the H as well as the permeability of the material the magnetic flux is passing through.
- Note that magnetic flux density (B) is also often simply called the magnetic field.
- The two are related by the equation: $B = \mu H$.
- **Permeability** (μ) indicates how easily magnetic flux can pass through the material.
- Magnetic materials increase μ so more B is generated per H within the coil.



$$H = \frac{F_\mathrm{m}}{\ell}$$

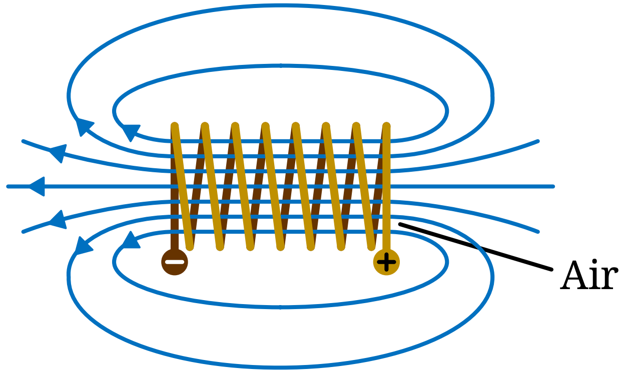
$$B = \mu H$$



- Magnetic flux in air or in a magnetic core

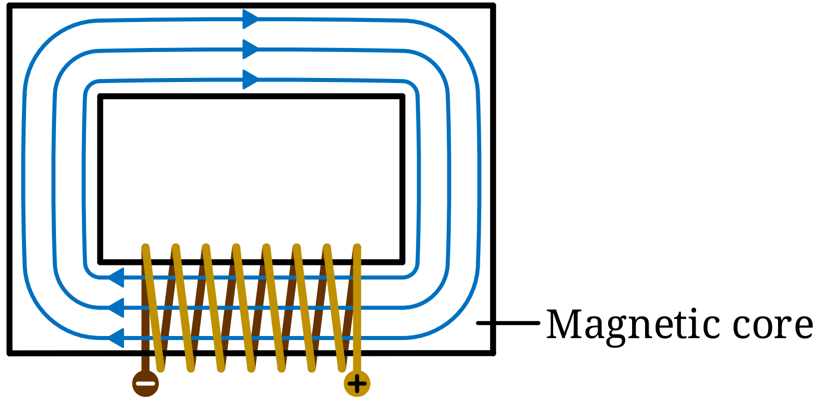
Magnetics – permeability

- The permeability can be broken up into two parts:
- **<u>Permeability of vacuum</u>** (μ_0) is the 'reference' permeability for magnetic field.
- $\mu_0 = 4\pi \times 10^{-7} = 1.2566370614 \times 10^{-6}$ Hm^{-1}
- If magnetic flux only travelled in vacuum, $B = \mu_0 H$.
- **<u>Relative permeability</u>** (μ_r) is a 'multiplier' that improves overall permeability for the magnetic field.
- For example, air has μ_r of 1 while typical ferrite core has μ_r of 1000 to 3000.
- So the relationship between B and H becomes:
- $B = \mu_0 \mu_r H$



$$H = \frac{F_m}{\ell}$$

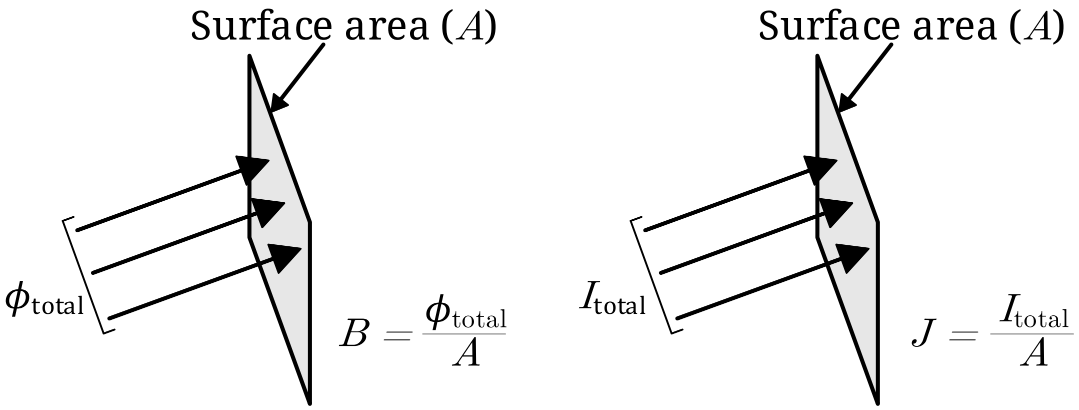
$$B = \mu H$$



- Magnetic flux in air or in a magnetic core

Magnetics – magnetic flux

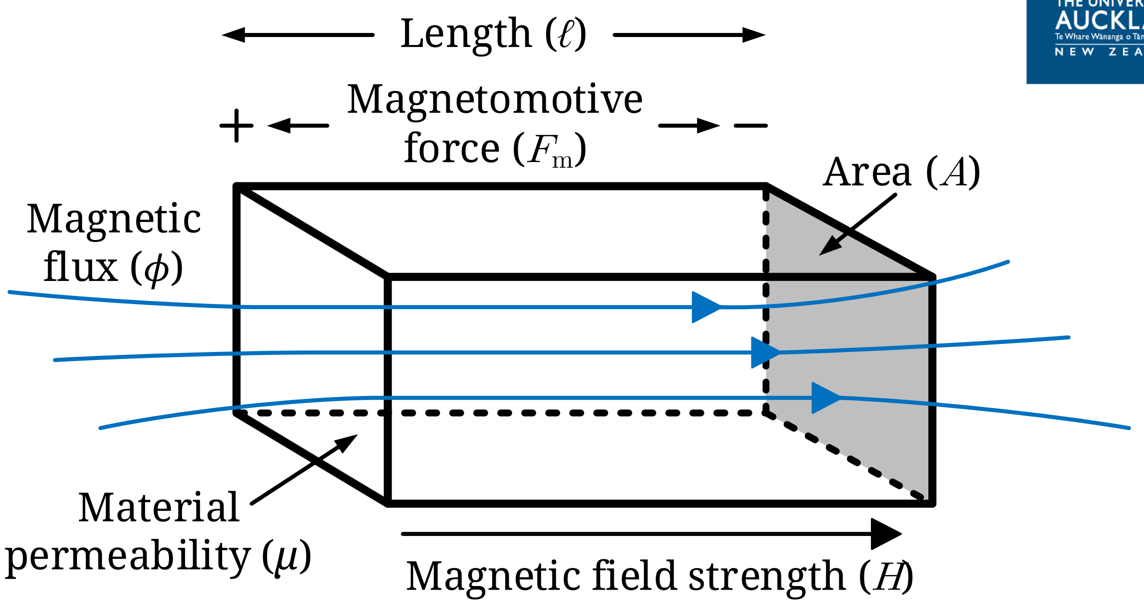
- If F_{m} is analogous to V, **magnetic flux** (ϕ) is analogous to electric current (I).
- If magnetic flux passes through a surface with an area of A, the **magnetic flux density** (B) can be found given:
- $\phi = \int_S B \cdot dA$
- If B is assumed to be uniform, the equation can be simplified to:
- $\phi = BA$
- Similarly, uniform current passing through a surface with an area of A results in current density (J).
- $I = JA$

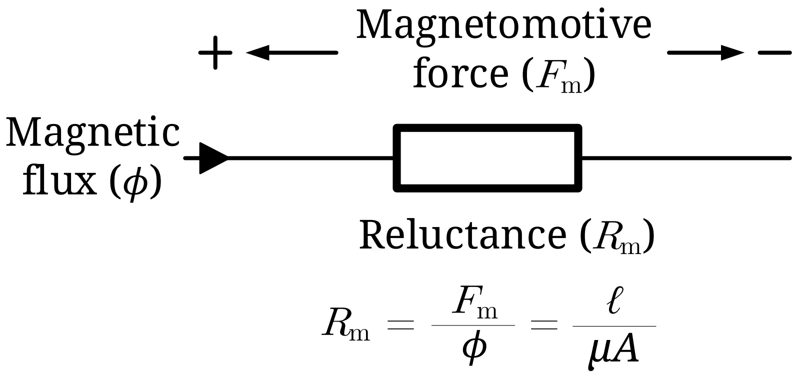


- Magnetic flux and electric current

Magnetics – reluctance

- Given that $H = \frac{B}{\mu}$ and $B = \frac{\phi}{A}$,
- $F_\mathrm{m} = \frac{B}{\mu}\ell = \frac{\ell}{\mu A}\phi$
- **Reluctance** (R_m) is given by: $R_\mathrm{m} = \frac{\ell}{\mu A}$.
- $\therefore F_\mathrm{m} = \phi R_\mathrm{m}$
- Similar to Ohm's law: $V = IR$.
- $R = \frac{\ell}{\sigma A}$,
- σ is the electrical conductivity.



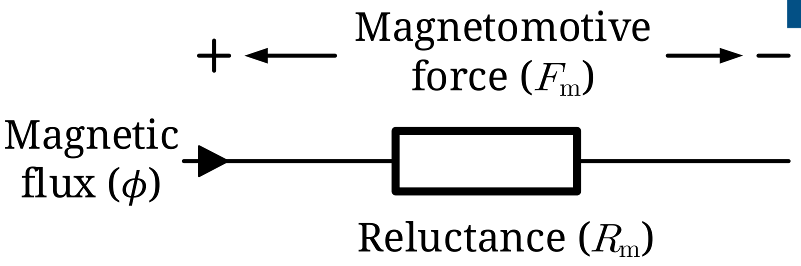


Magnetics – magnetic circuits and electric circuits

- Many of the terms between magnetic circuits and electrical circuits are quite similar with each other.

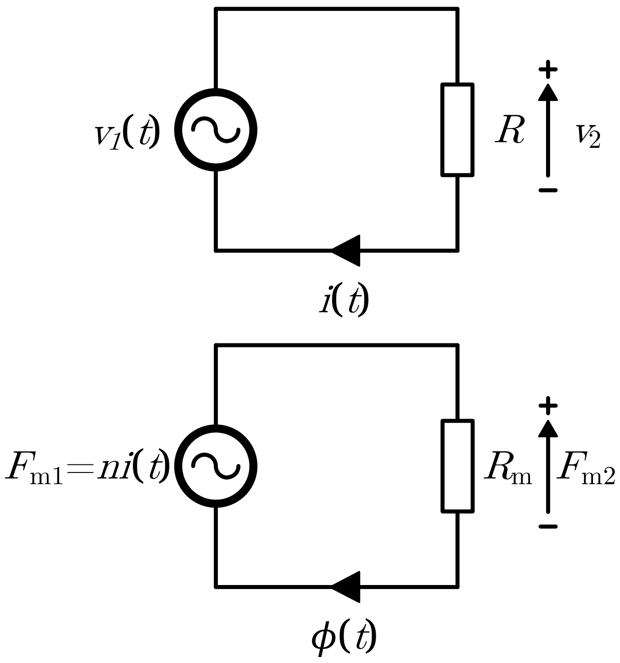
Magnetic quantity	Electric quantity
Magnetomotive force (F_m)	Electromotive force or voltage (V)
Magnetic field strength (H)	Electric field strength (E)
Magnetic flux (ϕ)	Current (I)
Magnetic flux density (B)	Current density (J)
Reluctance (R_m)	Resistance (R)
Permeability (μ)	Conductivity (σ)

- Analogous terms between magnetic and electric circuits



$$R_m = \frac{F_m}{\phi} = \frac{\ell}{\mu A}$$

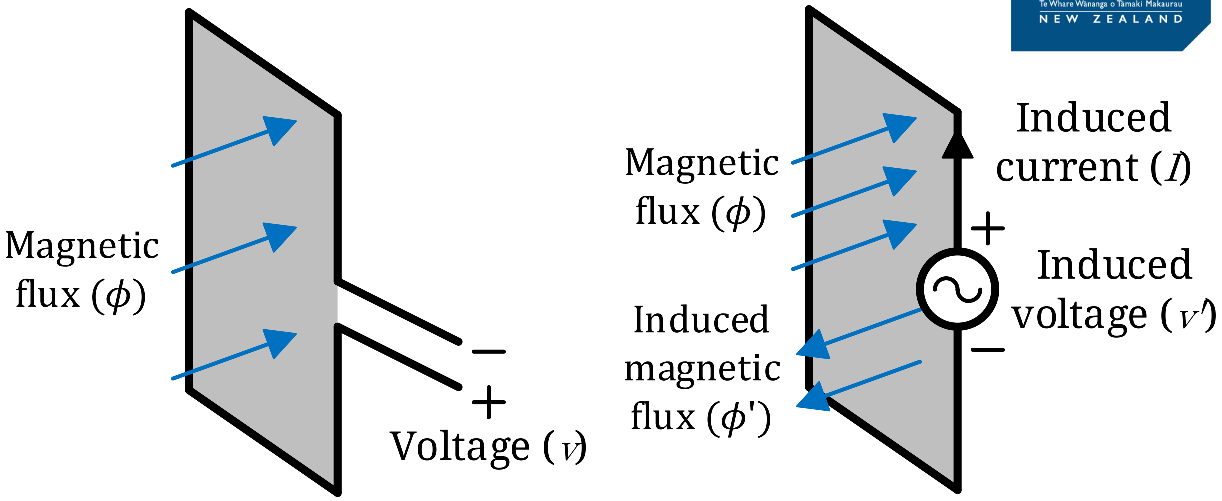
- Reluctance in magnetic circuits



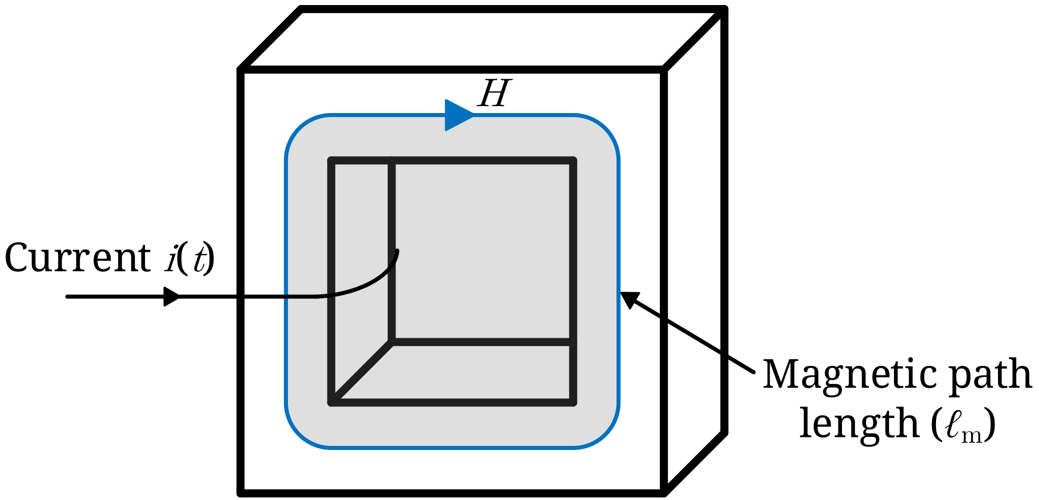
- Comparison of electric and magnetic circuits

Magnetics – laws

- **Faraday's Law**: $v(t) = \frac{d\phi(t)}{dt}$
- If B is assumed to be uniform, $v(t) = A\frac{dB(t)}{dt}$.
- Voltage induced in a winding is dependent on the change of magnetic field passing through the cross-sectional area of the loop.
- **Lenz's Law**: $v'(t) = -A\frac{dB(t)}{dt}$
- The induced voltage has a polarity that counteracts the magnetic field into the loop.
- **Ampere's Law**: $\int_S H \cdot d\ell$ = total current enclosed in the path.
- Given uniform magnetic field, $F_\mathrm{m}(t) = H(t)\ell_\mathrm{m} = i(t)$.
- Net magnetomotive force around a closed loop is equal to the total current passing through the path.



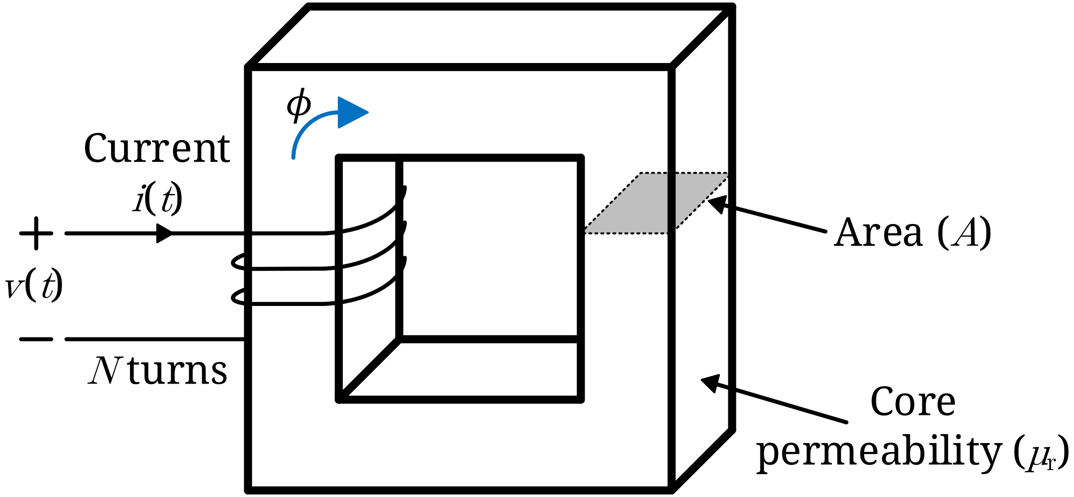
- Magnetic flux into open and closed loops



- A current inducing magnetic field within a magnetic material

Magnetics – inductance

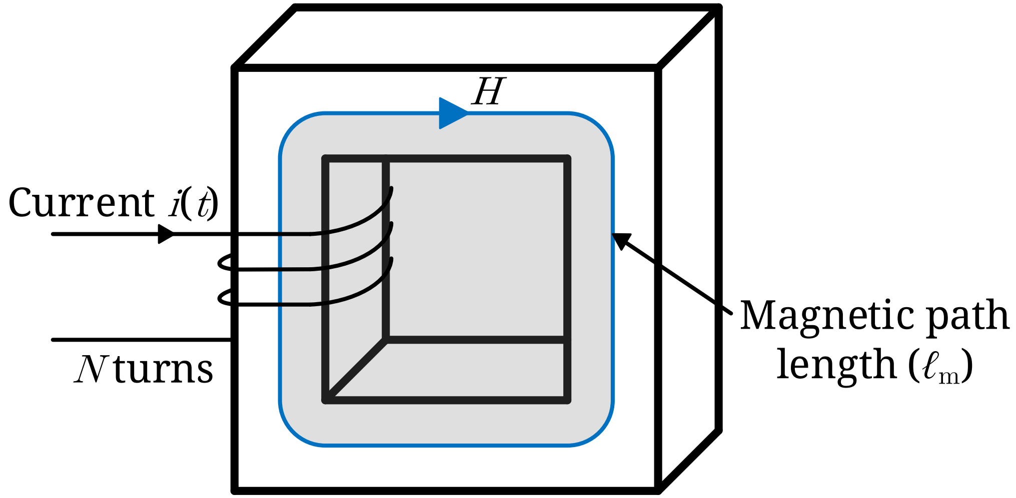
- From Faraday's Law, the voltage in each turn of wire is:
- $v_{\text{turn}}\ (t) = \frac{d\phi(t)}{dt}$
- The same flux passes through each turn of wire so total voltage across the winding is given by:
- $v(t) = Nv_{\text{turn}}\ (t) = N\frac{d\phi(t)}{dt}$
- The average magnetic field within the winding is:
- $v(t) = NA\frac{dB(t)}{dt}$
- Given that $B = \ \mu_0\mu_{\text{r}}H$,
- $v(t) = \mu_0\mu_{\text{r}}NA\frac{dH(t)}{dt}$



- Multiple turns of wire forming an inductor around the magnetic core

Magnetics – inductance

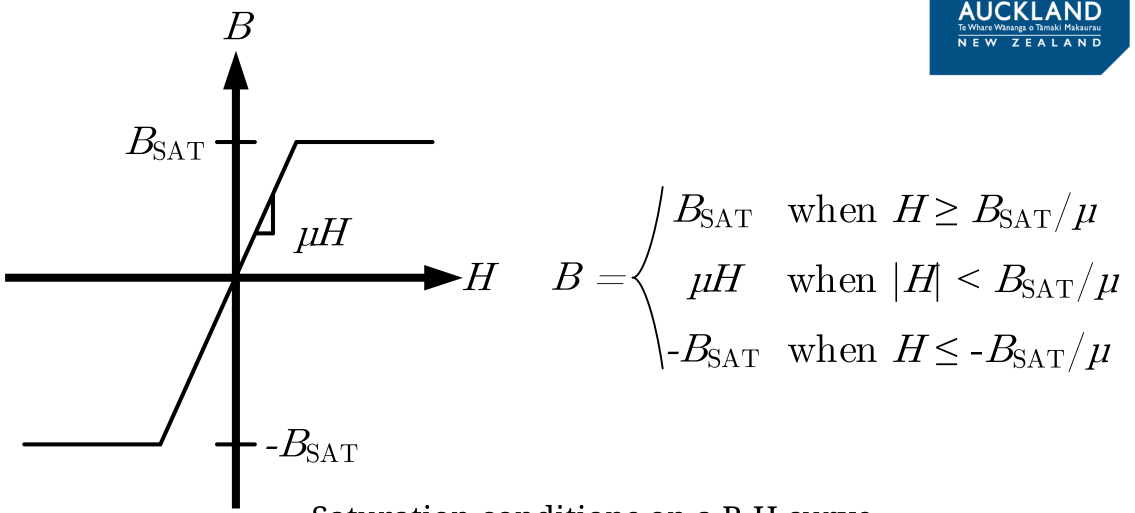
- Ampere's Law: $H(t)\ell_{\mathrm{m}} = i(t)$
- The number of turns multiply the current in the enclosed area.
- $\therefore H(t)\ell_{\mathrm{m}} = Ni(t)$
- $H(t) = \frac{Ni(t)}{\ell_{\mathrm{m}}}$
- Since $v(t) = \mu_0\mu_{\mathrm{r}}NA\frac{dH(t)}{dt}$,
- $v(t) = \frac{\mu_0\mu_{\mathrm{r}}N^2A}{\ell_{\mathrm{m}}}\frac{di(t)}{dt}$
- **Inductance** (L) is defined as $L = \frac{\mu_0\mu_{\mathrm{r}}N^2A}{\ell_{\mathrm{m}}}$ so:
- $v(t) = L\frac{di(t)}{dt}$
- Inductance resists the change in the current inside of a coil.



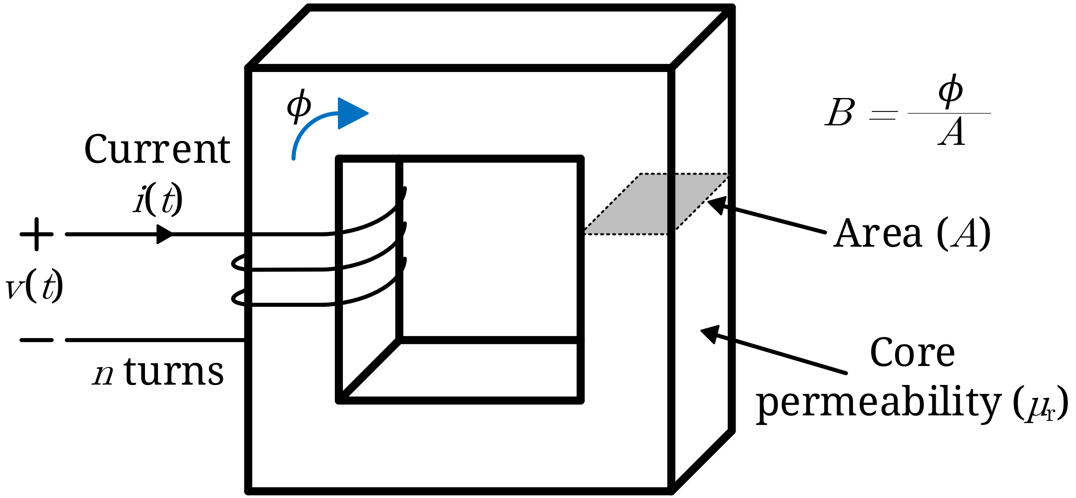
- Ampere's Law for multiple turn windings.

Magnetics – core saturation

- Magnetic materials cannot pass infinite amount of magnetic field.
- The B-H curve describes the amount of B within the core for a given amount of H.
- At saturation, $B = B_{SAT} \therefore \frac{dB(t)}{dt} = 0$
- This sets the Faraday's Law to be:
- $v(t) = nA\frac{dB(t)}{dt} = 0$
- $\therefore v(t) = L\frac{di(t)}{dt} = 0$
- As no amount of voltage is induced between the terminals, the inductor behaves like a short circuit under core saturation.
- This shows that the behaviour of the inductor only holds when the core is not saturated.



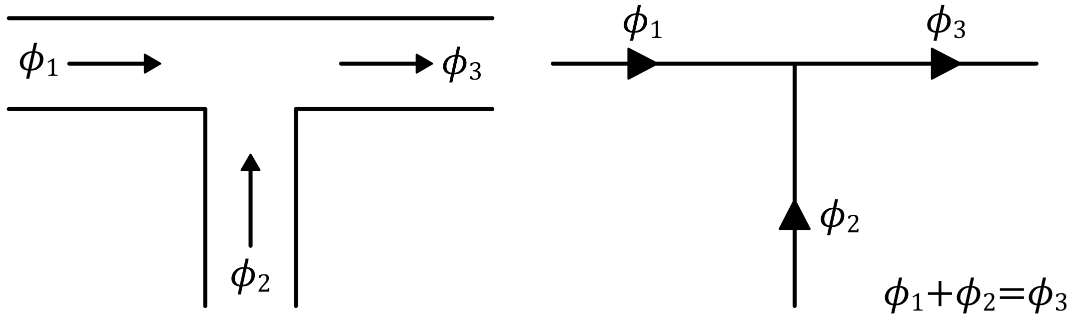
- Saturation conditions on a B-H curve



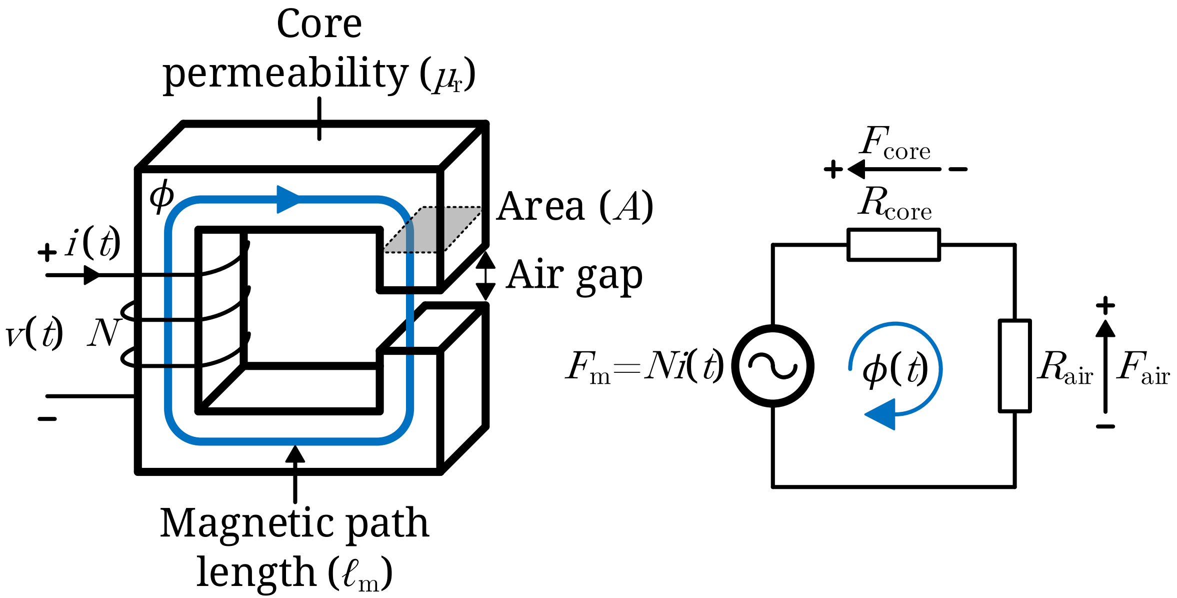
- An example inductor wound around a magnetic core

Magnetics – magnetic circuits

- All magnetic flux going into the node equal to the magnetic flux going out of the node (like KCL):
- $\phi_1 + \phi_2 = \phi_3$
- Similarly, sum of the magnetomotive force in a loop is equal to zero (like KVL):
- $F_\mathrm{m} = F_\mathrm{core} + F_\mathrm{air}$
- Given that the core has an air gap, the reluctances are:
- $R_\mathrm{core} = \frac{\ell_\mathrm{core}}{\mu_0 \mu_\mathrm{r} A}$ and $R_\mathrm{air} = \frac{\ell_\mathrm{air}}{\mu_0 A}$
- $R_\mathrm{e} = R_\mathrm{core} + R_\mathrm{air} = \frac{\ell_\mathrm{core}}{\mu_0 \mu_\mathrm{r} A} + \frac{\ell_\mathrm{air}}{\mu_0 A} = \frac{\ell_\mathrm{core}}{\mu_0 \mu_\mathrm{r} A} + \frac{\ell_\mathrm{air} \mu_\mathrm{r}}{\mu_0 \mu_\mathrm{r} A}$
- $= \frac{\ell_\mathrm{core} + \ell_\mathrm{air} \mu_\mathrm{r}}{\mu_0 \mu_\mathrm{r} A} = \frac{\frac{\ell_\mathrm{core}}{\mu_\mathrm{r}} + \ell_\mathrm{air}}{\mu_0 A}$
- Here, R_e is the effective reluctance of the circuit.



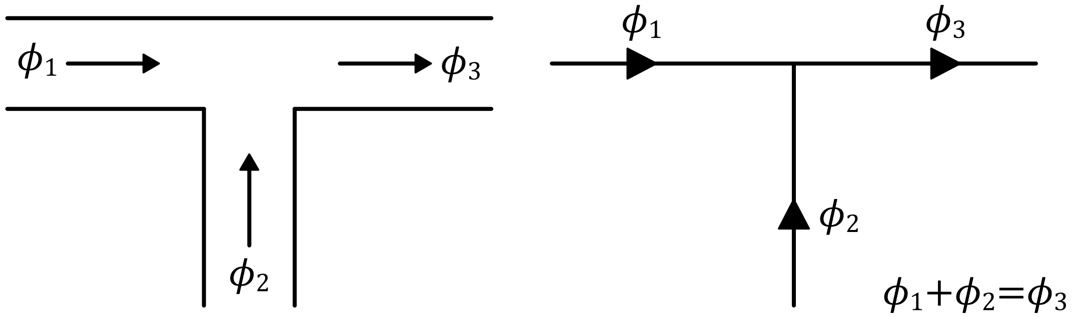
- Node analysis for magnetic circuit



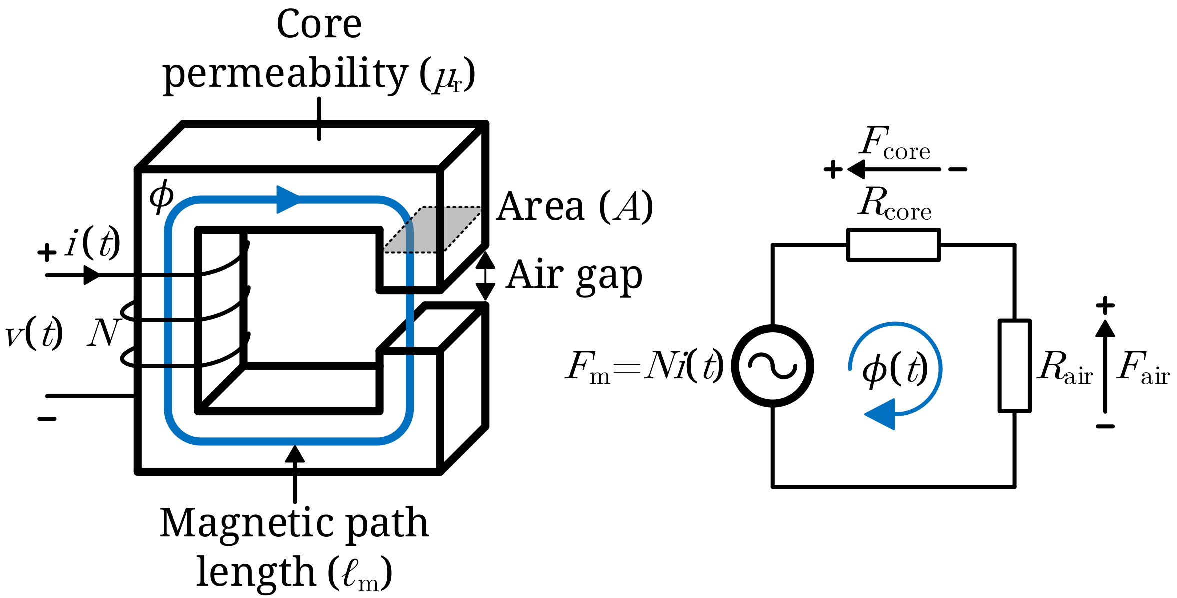
- Magnetic circuit of an inductor wound on a core

Magnetics – magnetic circuits

- From Ampere's Law:
- $F_{\mathrm{m}} = NI = \phi(R_{\mathrm{core}} + R_{\mathrm{air}}) = \phi R_{\mathrm{e}}$
- $\therefore \phi = \frac{NI}{R_{\mathrm{e}}}$
- A higher reluctance decreases the magnetic flux flowing in the magnetic circuit.
- Since $R_{\mathrm{e}} = \frac{\ell}{\mu_0 \mu_{\mathrm{e}} A}$ and $L = \frac{\mu_0 \mu_{\mathrm{e}} N^2 A}{\ell_{\mathrm{m}}}$,
- If μ_{e} is reduced due to higher R_{e}, the inductance of the winding decreases.



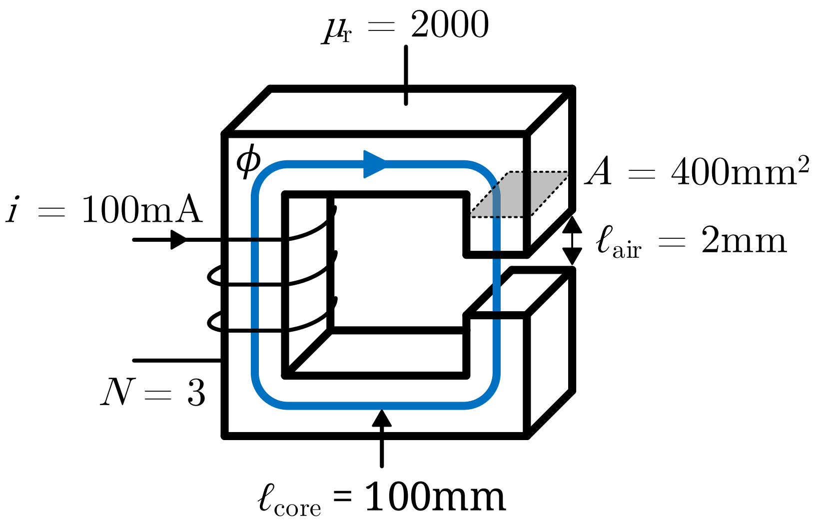
- Node analysis for magnetic circuit



- Magnetic circuit of an inductor wound on a core

Magnetics – gapped core

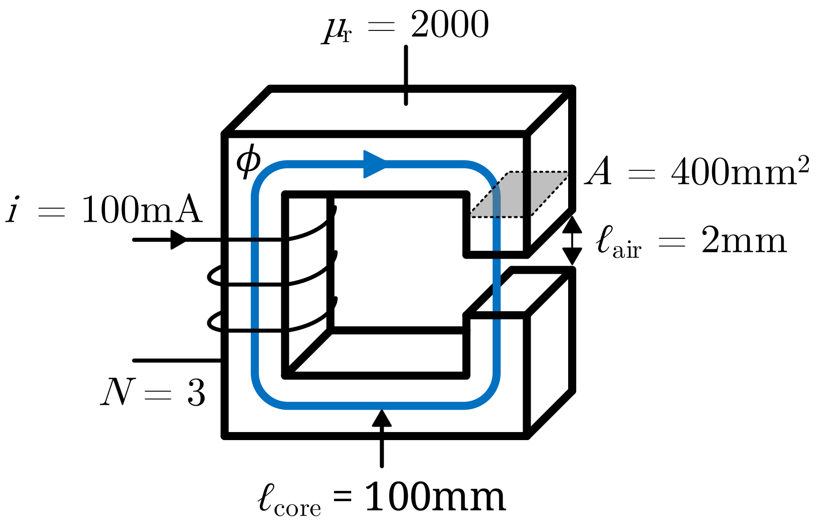
- What is the inductance of a winding around a gapped core?
- $R_{\mathrm{core}} = \frac{\ell_{\mathrm{core}}}{\mu_0 \mu_r A}$
- $R_{\mathrm{air}} = \frac{\ell_{\mathrm{air}}}{\mu_0 A}$
- $R_e = \frac{\frac{\ell_{\mathrm{core}}}{\mu_r} + \ell_{\mathrm{air}}}{\mu_0 A}$
- $L = \frac{\mu_0 \mu_e N^2 A}{\ell_m} = \frac{N^2}{R_e}$
- What is the effective permeability (μ_e) of the gapped core?
- $\mu_e = \frac{\ell_{\mathrm{core}} + \ell_{\mathrm{air}}}{\mu_0 R_e A}$



- Example inductor wound on a gapped core

Magnetics – gapped core

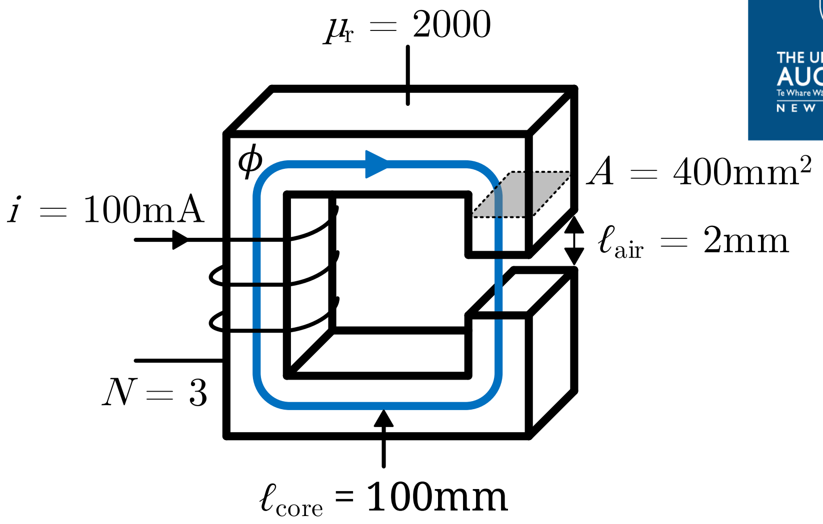
- What is the inductance of a winding around a gapped core?
- $R_{\text{core}} = \frac{\ell_{core}}{\mu_0 \mu_r A} = \frac{0.1}{4\pi \times 10^{-7} \times 2000 \times 0.0004} = 99472\ \text{H}^{-1}$
- $R_{\text{air}} = \frac{\ell_{air}}{\mu_0 A} = \frac{0.002}{4\pi \times 10^{-7} \times 0.0004} = 3978874\ \text{H}^{-1}$
- $L = \frac{\mu_0 \mu_e N^2 A}{\ell_m} = \frac{N^2}{R_e} = \frac{3^2}{99472 + 3978874} = 0.00000221 = 2.21\ \mu\text{H}$
- What is the effective permeability (μ_e) of the gapped core?
- $\mu_e = \frac{\ell}{\mu_0 R_e A} = \frac{0.102}{4\pi \times 10^{-7} \times 4078346 \times 0.0004} = 49.8$



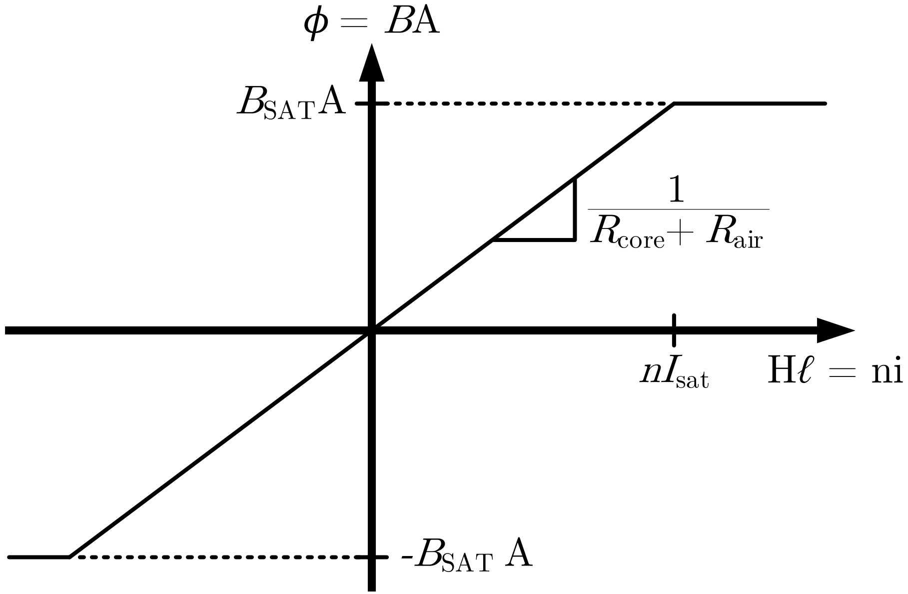
- Example inductor wound on a gapped core

Magnetics – gapped core

- What is the saturation current (I_{SAT}) if B_{SAT} = 400 mT?
- $H\ell_m = NI$
- $B = \mu_0 \mu_r H$
- $I = \frac{H\ell_m}{N} = \frac{B\ell_m}{N\mu_0\mu_r}$
- Effective reluctance: $R_e = \frac{\ell}{\mu_e A}$,
- $I = \frac{H\ell_m}{N} = \frac{B\ell_m}{N\mu_0\mu_e} = \frac{BA}{N} R_e$



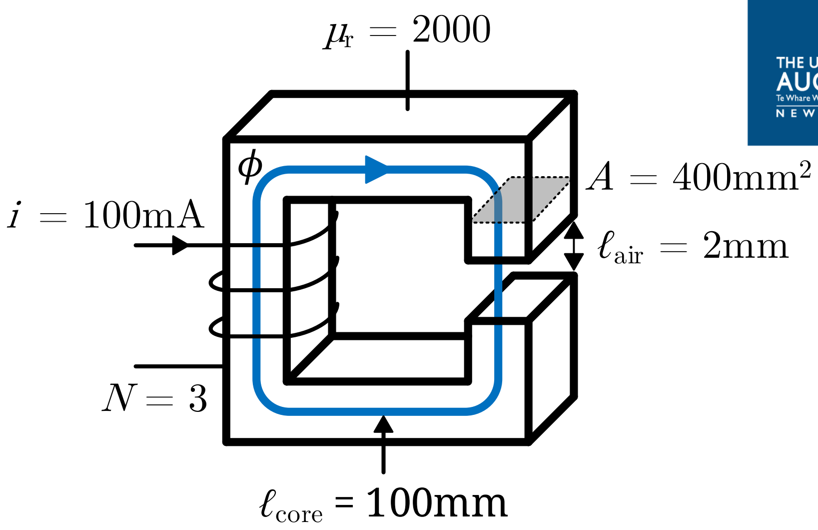
- Example inductor wound on a gapped core



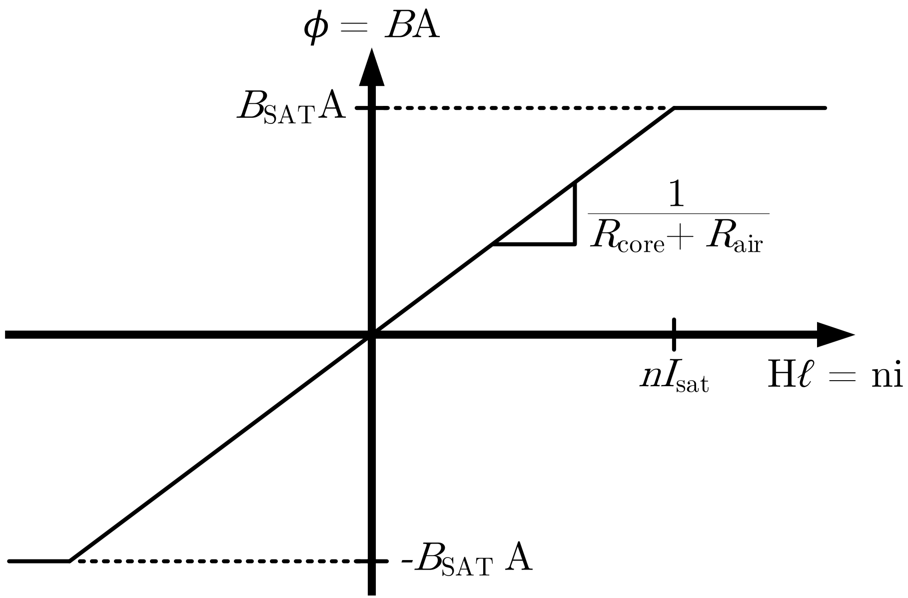
- B-H curve for inductor wound on a gapped core

Magnetics – gapped core

- What is the saturation current (I_{SAT}) if $B_{SAT} = 400$ mT?
- $H\ell_m = NI$
- $B = \mu_0 \mu_r H$
- $I = \frac{H\ell_m}{N} = \frac{B\ell_m}{N\mu_0\mu_r}$
- Effective reluctance: $R_e = \frac{\ell}{\mu_e A}$,
- $I = \frac{H\ell_m}{N} = \frac{B\ell_m}{N\mu_0\mu_e} = \frac{BA}{N}R_e$
- $I_{sat} = \frac{B_{SAT}A}{N}R_e = \frac{0.4\times 0.0004}{3}4078346 = 218$ A



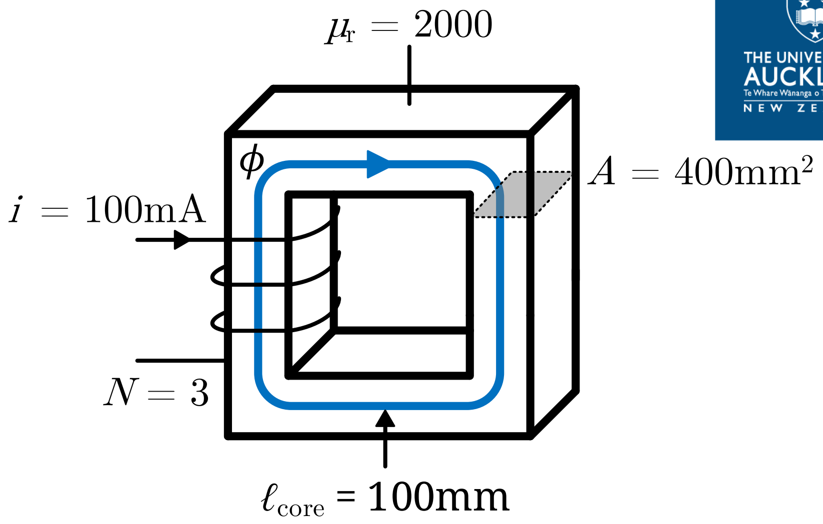
- Example inductor wound on a gapped core



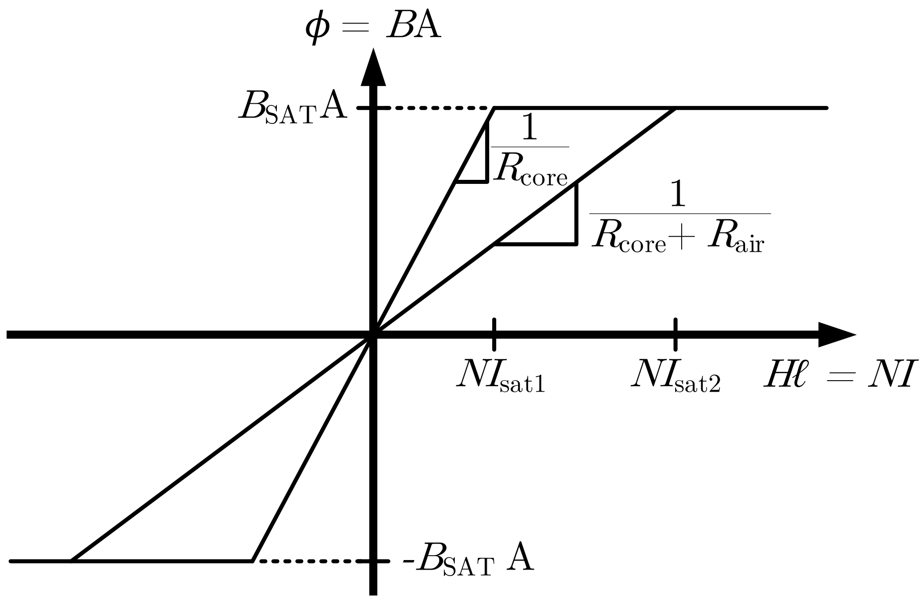
- B-H curve for inductor wound on a gapped core

Magnetics –core without air gap

- What is the inductance if the core had no air gap?
- $R_{\text{core}} = \frac{\ell_{core}}{\mu_0 \mu_r A} = \frac{0.102}{4\pi \times 10^{-7} \times 2000 \times 0.0004} = 101461\ \text{H}^{-1} = 1 \times 10^5\ \text{H}^{-1}$
- $L = \frac{3^2}{101461} = 88.70\ \mu\text{H}$
- Using the same conditions as before, if $B_{\text{SAT}} = 400$ mT,
- $i_{\text{sat}} = \frac{B_{\text{SAT}} A}{N} R_{\text{core}} = \frac{0.4 \times 0.0004}{3} \times 101461 = 5.41$ A
- Remember that 2mm air gap created reluctance of about $4 \times 10^6\ \text{H}^{-1}$.
- A small air gap can form a large reluctance in the magnetic flux path.
- The high inductance is an indicator of the winding inducing a large magnetic flux per input current.
- However, the core reaches B_{SAT} quickly if too much magnetic flux is generated.
- The air gap in the cores increase the effective reluctance to push the saturation point further away.



- Example inductor wound on a core without an air gap



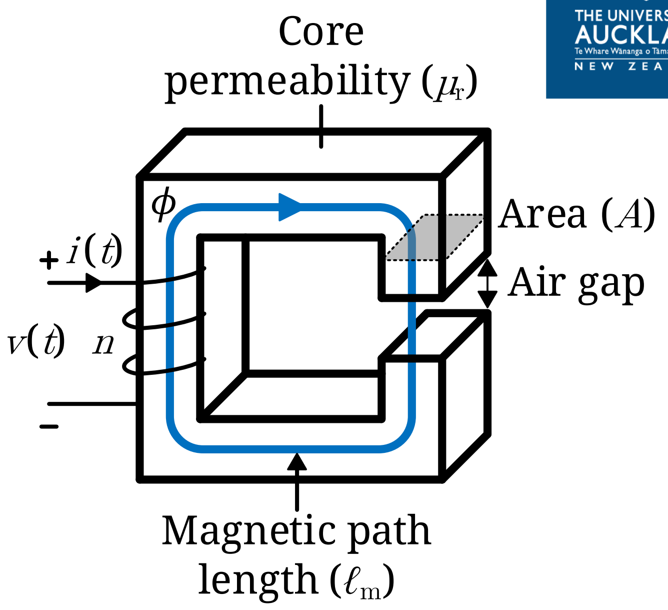
- B-H curves for inductor wound on a different cores

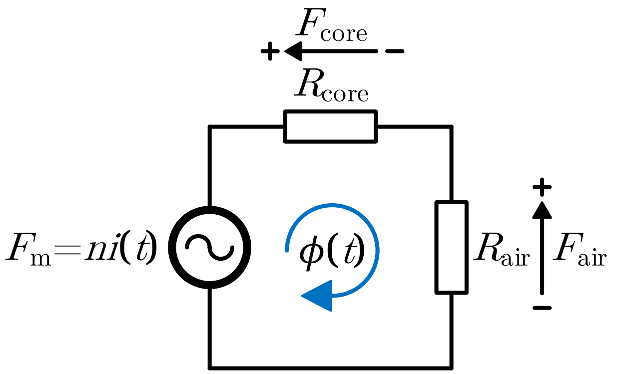
Magnetics – summary

- Magnetic circuits are analogous to electric circuits

Magnetic quantity	Electric quantity
Magnetomotive force (F_{m})	Electromotive force or voltage (V)
Magnetic field strength (H)	Electric field strength (E)
Magnetic flux (ϕ)	Current (I)
Magnetic flux density (B)	Current density (J)
Reluctance (R_{m})	Resistance (R)
Permeability (μ)	Conductivity (σ)

- **<u>Faraday's Law</u>**: $v(t) = \frac{d\phi(t)}{dt}$
- **<u>Lenz's Law</u>**: $v'(t) = -A\frac{dB(t)}{dt}$
- **<u>Ampere's Law</u>**: $\int_S H \cdot d\ell$
- Magnetic circuits can be formed using F_{m}, ϕ and R_{m}.
- Magnetic materials saturate and air gaps improve current carrying capacity by moving the saturation point.





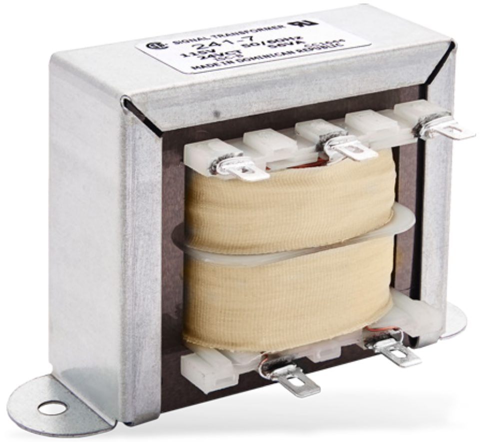
- Magnetic circuit of an inductor wound on a core with an air gap

Magnetics

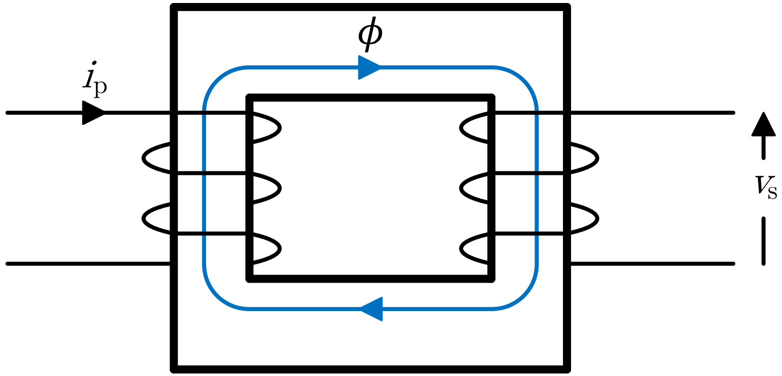
Ideal transformer

Magnetics – transformer

- Transformers are used in a wide range of applications to step the voltages up and down as well as for isolation purposes.
- A transformer transfers energy using magnetic flux without any conductive connection.
- According to Ampere's Law, electric current (i_p) generates a magnetic field strength of H.
- **Ampere's Law**: $H(t) = \frac{Ni(t)}{\ell_\mathrm{m}}$
- Within the core, magnetic field given by: $B = \mu_0 \mu_\mathrm{r} H$
- According to Faraday's Law, an electromotive force is induced in the secondary coil.
- **Faraday's Law**: $v(t) = NA\frac{dB(t)}{dt} = N\frac{d\phi(t)}{dt}$
- Since only changes in magnetic flux induce voltage in the secondary, DC input does not induce any output.



- Transformers [3] and [4]

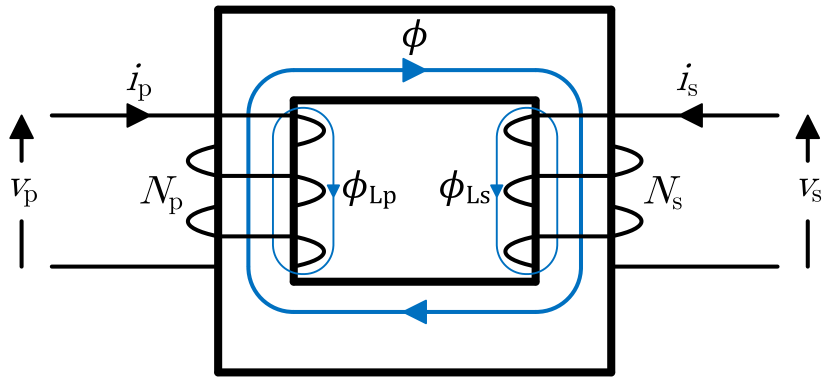


- Current into the primary coil inducing voltage in the secondary coil

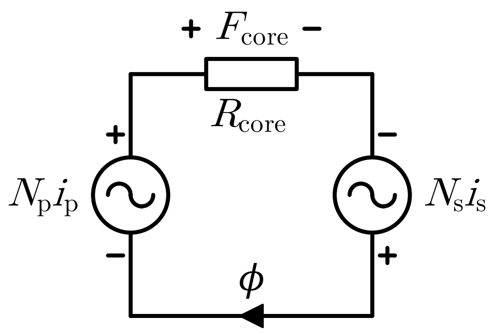
[3] https://nz.mouser.com/new/bel-signal-transformer/signal-transformer-two-4-one-power-transformers/
[4] https://the-rsgroup.com/the-1000th-power-transformer/

Magnetics – ideal transformer

- An ideal transformer has:
  - No losses in the core and the winding
  - Infinite relative permeability in the core ($\mu_r = \infty$)
  - No leakage magnetic flux (ϕ_{L_p} and $\phi_{L_s} = 0$)
- Ideally, all magnetic flux generated by one coil passes through the other coil in a transformer.
- Leakage magnetic flux are magnetic flux generated by the energised coil that does not pass through the other coil.
- N_p and N_s refer to the number of turns in the primary coil and the secondary coil respectively.
- The magnetic circuit for a transformer gives:
- $R_{core} = \frac{\ell_{core}}{\mu_0 \mu_r A}$
- $F_{core} = N_p i_p + N_s i_s$
- $\phi R_{core} = N_p i_p + N_s i_s$



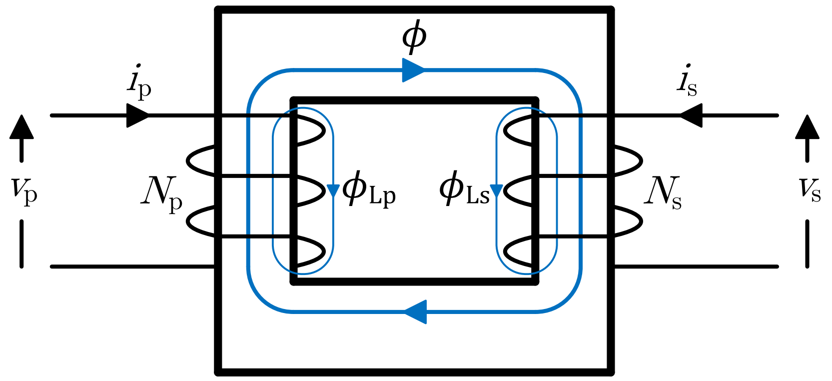
- An example ideal transformer



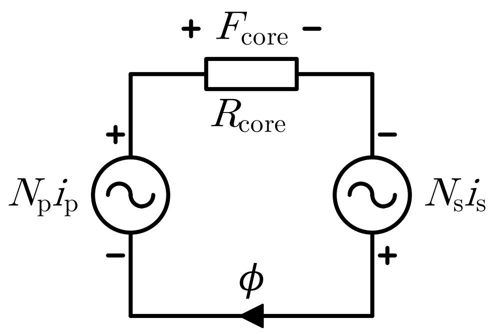
- Magnetic circuit for an ideal transformer

Magnetics – ideal transformer

- According to the magnetic circuit:
- $F_{\text{core}} = \phi R_{\text{core}}$
- In an ideal transformer, the reluctance of the core should be zero ($R_{\text{core}} = 0$) since $\mu_r = \infty$.
- $\therefore F_{\text{core}} = 0$
- Then the magnetic circuit becomes:
- $0 = N_p i_p + N_s i_s$
- Since Faraday's Law states that:
- $v_p = N_p \frac{d\phi(t)}{dt}$
- $v_s = N_s \frac{d\phi(t)}{dt}$
- Equate the two equations above by $\frac{d\phi(t)}{dt}$ to give:
- $\frac{d\phi(t)}{dt} = \frac{v_p}{N_p} = \frac{v_s}{N_s}$



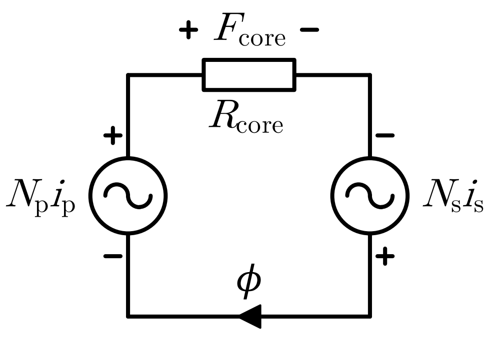
- An example ideal transformer

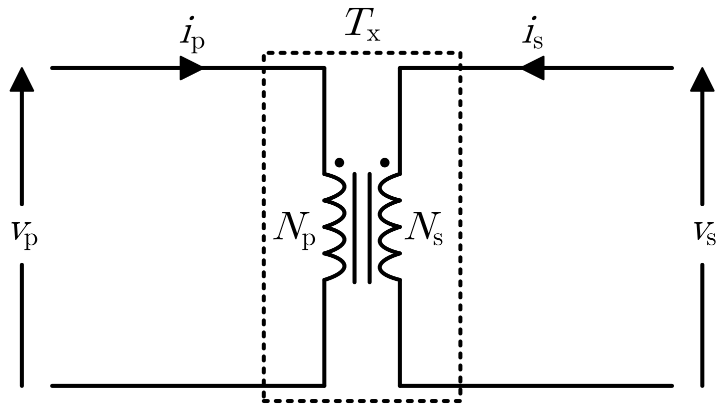


- Magnetic circuit for an ideal transformer

Magnetics – ideal transformer

- The previous equation can be modified to give:
- $\frac{v_p}{v_s} = \frac{N_p}{N_s}$
- **Turns ratio of the transformer** is defined as $\frac{N_p}{N_s}$.
- By modifying the turns ratio of the transformer, the secondary voltage can be stepped up or down.
- If the turns ratio is changed to modify the voltage, the current has to change accordingly.
- Given that an ideal transformer does not store any energy:
- $v_p i_p + v_s i_s = 0$
- $v_p i_p = -v_s i_s$
- Divide the equation using the previous equation $\frac{v_p}{N_p} = \frac{v_s}{N_s}$:
- $\frac{i_p}{N_p} = -\frac{i_s}{N_s}$
- $\frac{i_s}{i_p} = -\frac{N_p}{N_s}$

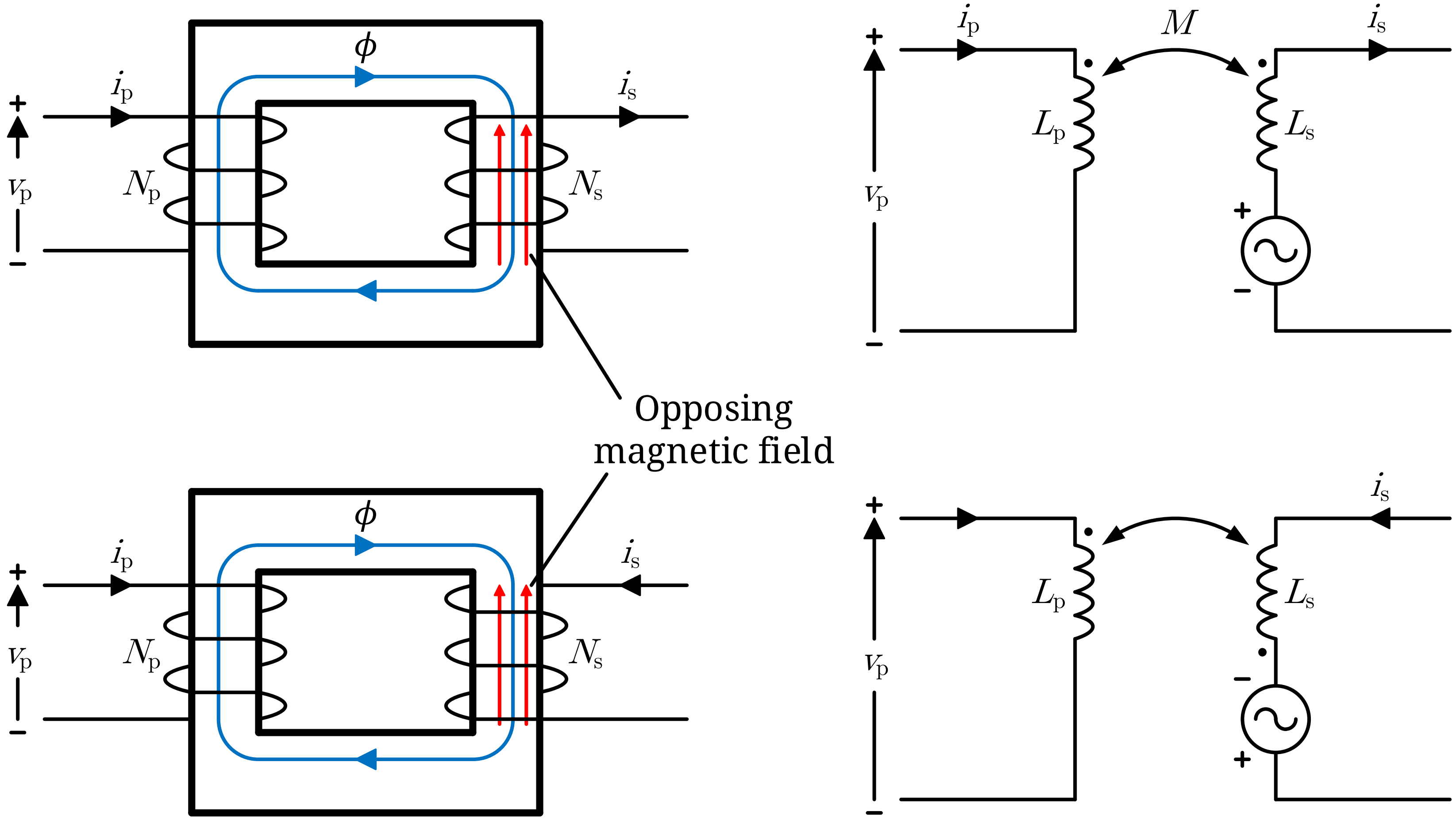




$$\frac{v_p}{v_s} = \frac{N_p}{N_s} \qquad \frac{i_s}{i_p} = -\frac{N_p}{N_s}$$

Magnetics – dot notation

- **Voltage going into the dot = voltage going out of the dot**

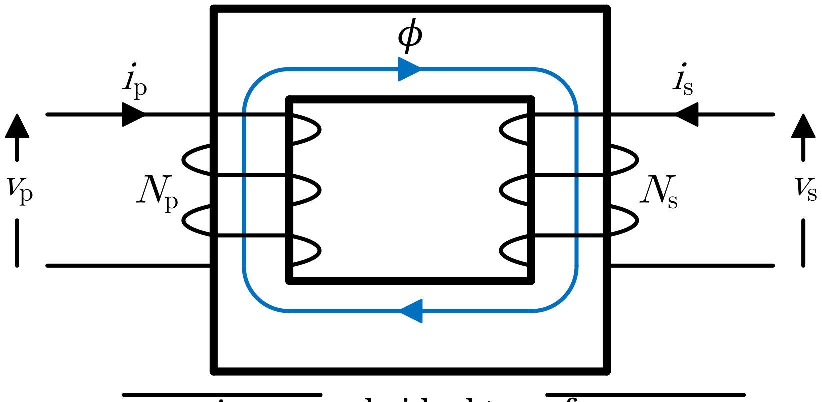


Magnetics

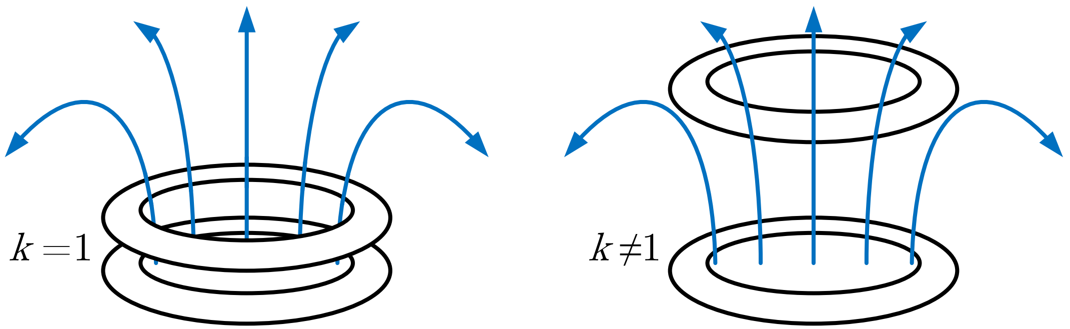
Practical transformer

Magnetics – coupling factor

- k is the **coupling factor** between the primary and secondary coils.
- k is defined to be $0 \leq k \leq 1$ and indicates the proportion of magnetic flux generated by the primary that passes through the secondary.
- In this case of an ideal transformer, all of the magnetic flux generated from the energised coil passes through the other coil ($k = 1$).



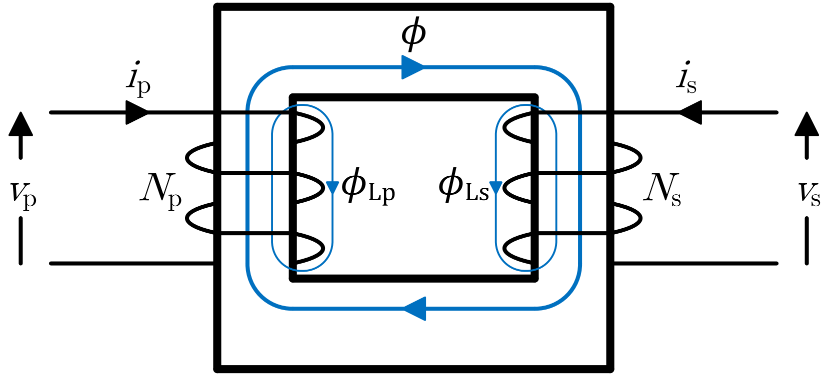
- An example ideal transformer



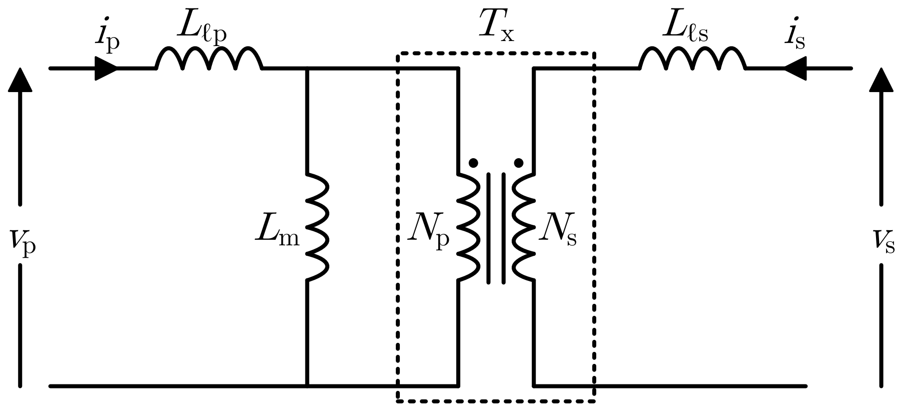
- Examples of high and low coupling factor

Magnetics – practical transformer

- In real life, a practical transformer has
  - Losses in the coils and the core
  - Finite relative permeability (μ_r) in the core
  - Leakage magnetic flux (ϕ_{L_p} and ϕ_{L_s}) $\neq 0$
  - Coupling factor (k) $\neq 1$.
- Infinite relative permeability does not exist in real life so some of the magnetic flux generated by the energised coil 'leaks out'.
- This leads the magnetic flux to be divided into two parts:
- **Magnetising magnetic flux (mutual flux)** (ϕ_m) passes through the core and is subject to saturation and core losses.
- **Leakage magnetic flux** (ϕ_{L_p} and ϕ_{L_s}) is generated by the energised coil, but does not reach the other coil.



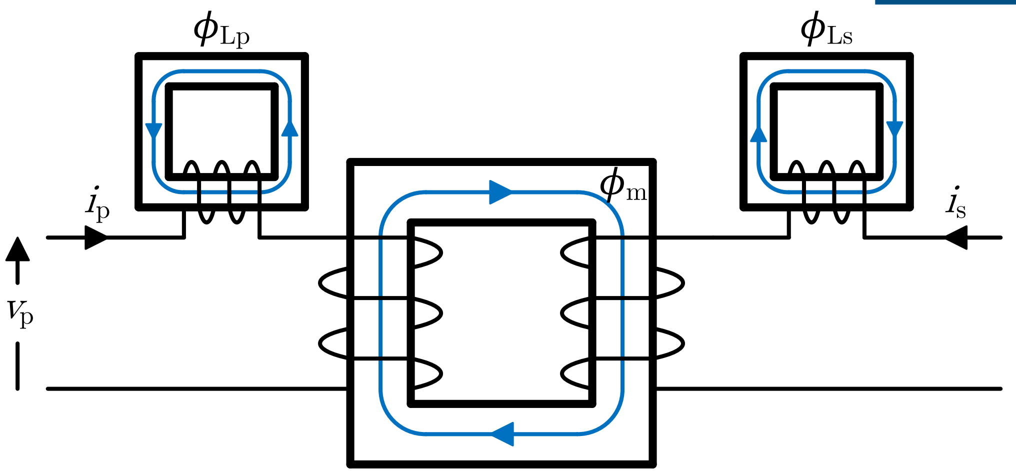
- An example transformer



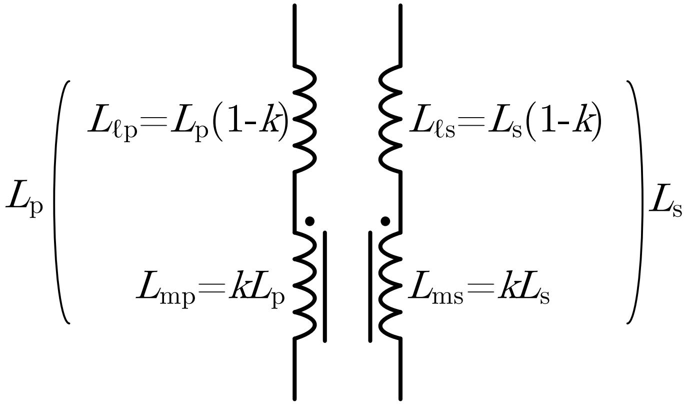
- Ideal transformer in circuit form with practical components

Magnetics – practical transformer

- If the magnetising inductance and leakage inductance could be separated out physically, they would be separate inductors wound in a magnetic core and air cores.
- **Magnetising inductance** (L_m) is due to magnetising magnetic flux (ϕ_m).
- **Leakage inductances** ($L_{\ell p}$ and $L_{\ell s}$) is due to the leakage magnetic flux (ϕ_{L_p} and ϕ_{L_s}).
- The leakage inductance adds to the total inductance, but does not help with transferring energy.
- In a practical transformer where $(k) \neq 1$,
- $L_{mp} = kL_p$ and $L_{ms} = kL_s$
- $L_{\ell p} = (1-k)L_p$ and $L_{\ell s} = (1-k)L_s$
- For a flyback converter, leakage inductances create issues such as ringing in the switches so should be minimised.



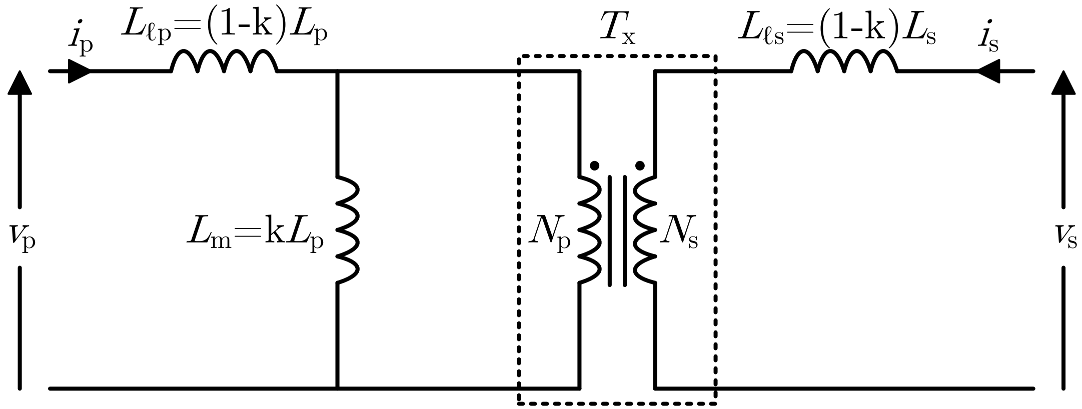
- Representation of separate magnetizing inductance and leakage inductances

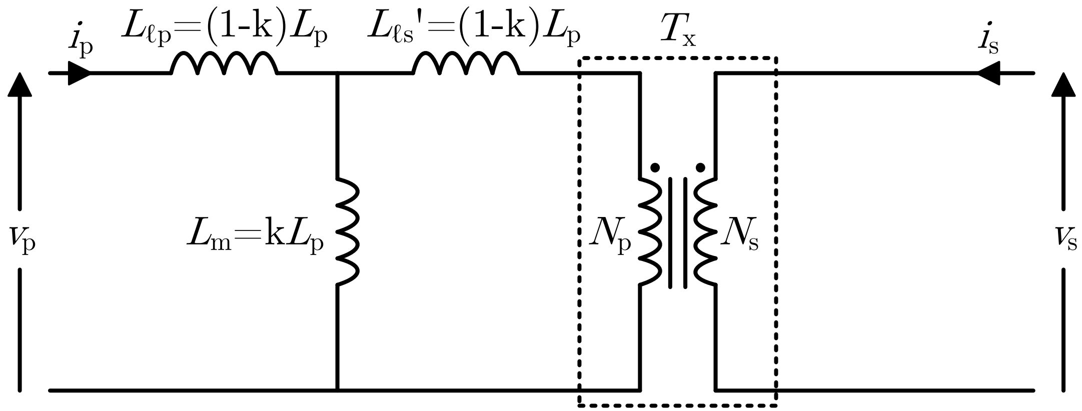


- Division of leakage and magnetising inductances in terms of coupling factor

Magnetics – T-model

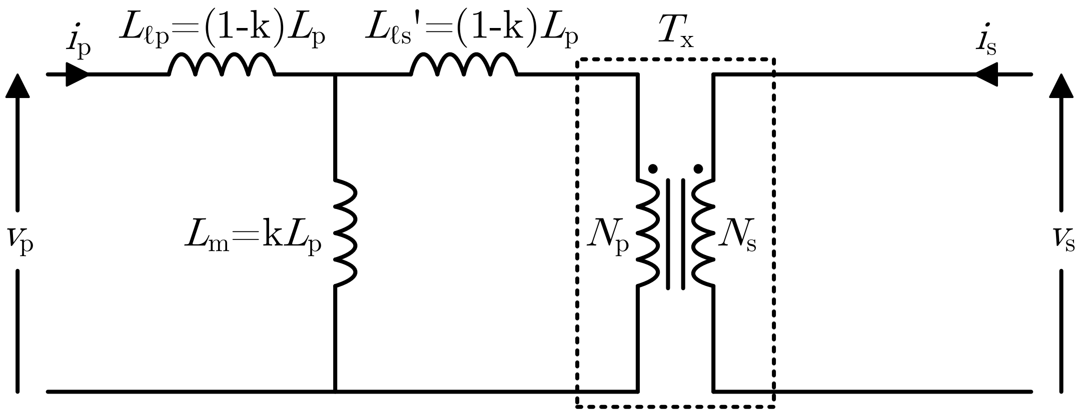
- To simplify the measurement of the transformer in real life, the leakage inductance in the secondary can be reflected on to the primary.
- The impedance transformation from secondary to primary is:
- $L'_{\ell s} = \left(\frac{N_p}{N_s}\right)^2 L_{\ell s} = \left(\frac{N_p}{N_s}\right)^2 (1-k)L_s$
- Given that $\sqrt{\frac{L_p}{L_s}} = \frac{N_p}{N_s}$, $L_s = L_p\left(\frac{N_s}{N_p}\right)^2$
- $L'_{\ell s} = \left(\frac{N_p}{N_s}\right)^2 (1-k)L_s = \left(\frac{N_p}{N_s}\right)^2 (1-k)L_p\left(\frac{N_s}{N_p}\right)^2$
- $\therefore L'_{\ell s} = (1-k)L_p$
- This is called the 'T-model' of a transformer.

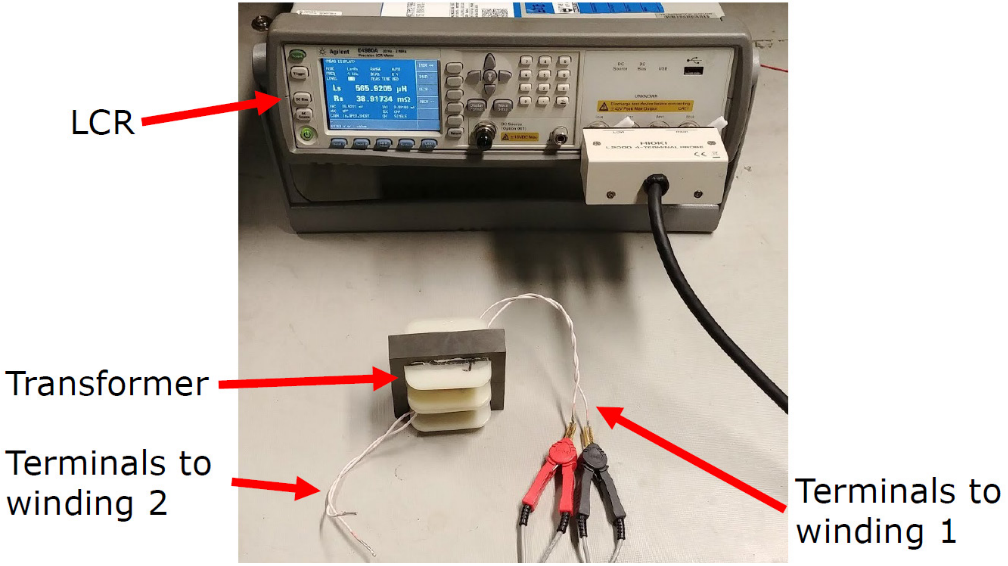




Magnetics – measurement

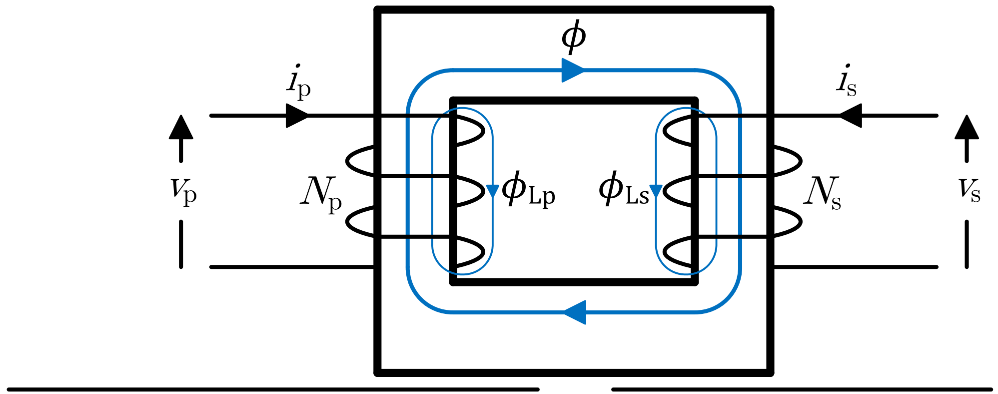
- Using an LCR metre, the inductance of the transformer can be measured.
- Open-circuit the secondary and measure the inductance as: $L_{\ell \mathrm{p}} + L_{\mathrm{m}} = (1-k)L_{\mathrm{p}} + kL_{\mathrm{p}} = L_{\mathrm{p}}$
- Short-circuit the secondary and measure the inductance as: L_{sc}
- Coupling factor is given by: $\sqrt{\frac{L_p - L_{sc}}{L_p}} = k$
- Using the turns ratio, the self-inductance of the secondary winding can be determined:
- $L_{\mathrm{s}} = \left(\frac{N_{\mathrm{p}}}{N_{\mathrm{s}}}\right)^2 L_{\mathrm{p}}$
- If turns ratio is unknown, measuring the inductance in the secondary with the primary open-circuited will also measure the secondary self-inductance.



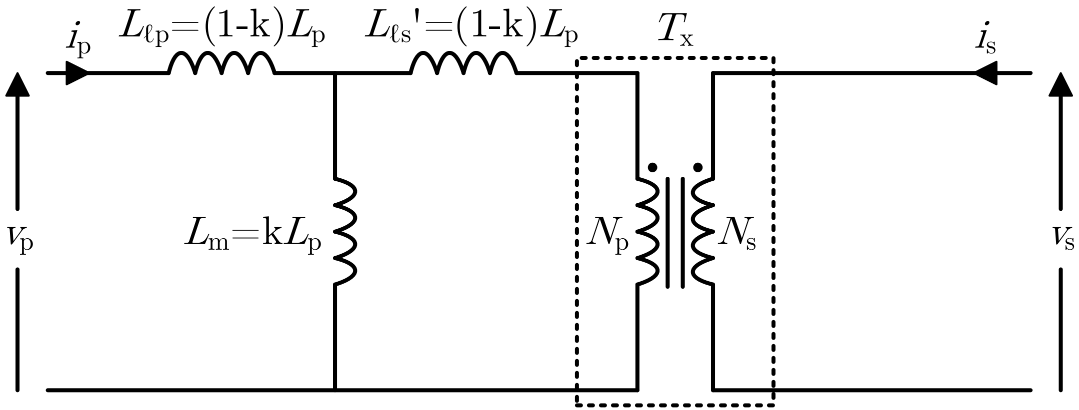


Transformers – summary

- In real life, a practical transformer has
  - Losses in the coils and the core
  - Finite relative permeability (μ_r) in the core
  - Leakage magnetic flux (ϕ_{L_p} and ϕ_{L_s}) $\neq 0$
  - Coupling factor (k) $\neq 1$.
- Turns ratio of the transformer is defined as $\frac{N_p}{N_s}$.
- Self-inductance is split into **<u>magnetising inductance</u>** and l**<u>eakage inductance</u>**.
- Coupling factor is a ratio measuring how much magnetic flux generated from one coil is passing through another coil.
- Dot notation:
  <u>VOLTAGE INTO THE DOT -> VOLTAGE OUT OF THE DOT</u>
- Use the LCR meter in the lab to find the transformer parameters.



- An example ideal transformer

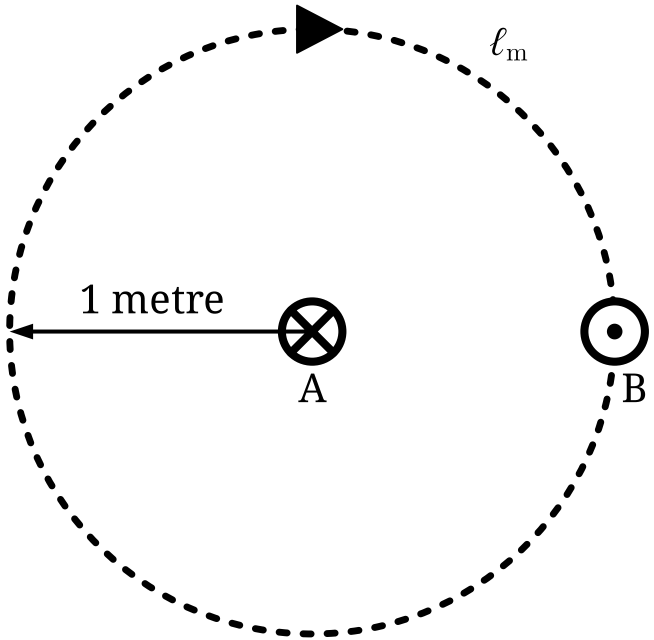


- T-model of a transformer

Appendix

Derivation of μ_0

- Two conductors are spaced 1 metre apart from each other. The conductors have 1 A of current flowing in opposing directions.
- Then the magnetic field strength due to conductor A at a distance of 1 metre is:
- $H = \frac{I}{\ell_\mathrm{m}} = \frac{1}{2\pi}$ Am^{-1}
- Here, ℓ_m is the closed circular path around the conductor.
- Force exerted on a conductor one metre away per ampere is defined as 2×10^{-7} Nm^{-1}.
- Assuming the conductors have the same length, the magnetic field can be found as:
- $B = \frac{F}{I} = \frac{2\times10^{-7}}{1}$T
- Then $\mu_0 = \frac{B}{H} = 2 \times 10^{-7} \times 2\pi = 4\pi \times 10^{-7}$ Hm^{-1}



- Two parallel conductors with currents flowing in opposing directions.

Relationship between mutual inductance and magnetising inductance

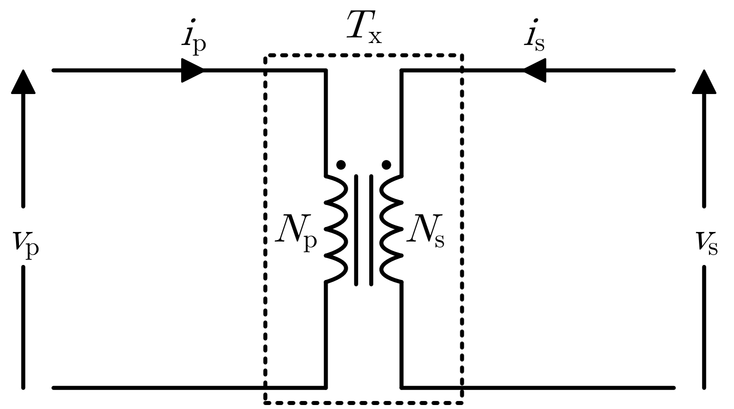
- **Inductance** (L) is defined as $L = \frac{\mu_0 \mu_r n^2 A}{\ell_m}$ so:
- $\frac{L_p}{L_s} = \frac{\frac{\mu_0 \mu_r N_p{}^2 A}{\ell_m}}{\frac{\mu_0 \mu_r N_s{}^2 A}{\ell_m}} = \left(\frac{N_p}{N_s}\right)^2$
- $\therefore \sqrt{\frac{L_p}{L_s}} = \frac{N_p}{N_s}$
- Mutual inductance is defined as: $M = k\sqrt{L_p L_s}$
- $M\frac{1}{L_p} = k\sqrt{L_p L_s}\frac{1}{L_p}$
- $M\frac{1}{L_p} = k\sqrt{\frac{L_s}{L_p}} = k\frac{N_s}{N_p}$
- $M = kL_p\frac{N_s}{N_p}$
- Since $L_{mp} = kL_p$
- $M = \frac{N_s}{N_p}L_{mp}$
- $\therefore L_{mp} = \frac{N_p}{N_s}M$
- L_{ms} is L_{mp} reflected on to the secondary so multiplied by $\left(\frac{N_s}{N_p}\right)^2$
- $L_{ms} = \left(\frac{N_s}{N_p}\right)^2\frac{N_p}{N_s}M = \frac{N_s}{N_p}M$

Magnetics

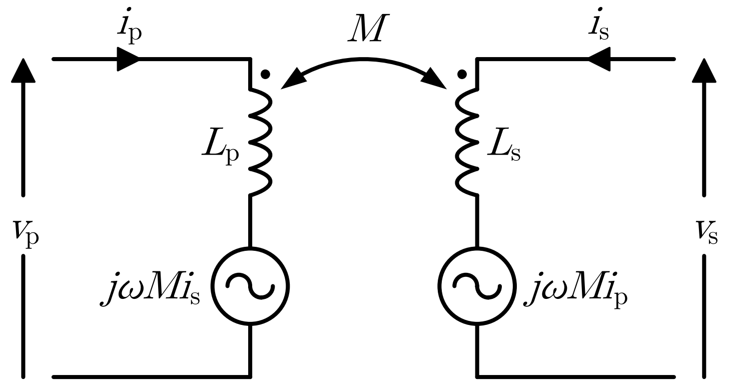
Ideal transformer

Magnetics – ideal transformer

- The ideal transformer in the circuit can be further broken down two electrically separated circuits.
- Previously Faraday's Law was derived to find:
- $v(t) = L\frac{di(t)}{dt}$
- In frequency domain, this is re-written as: $v = j\omega L i$
- ω is the **angular frequency** given by $\omega = 2\pi f$.
- Here, f is the frequency of the input waveform.
- ωL gives the reactance of the inductor (X_L).
- The two electrically separated coils are magnetically connected by the **mutual inductance** (M).
- Mutual inductance can be used to find the voltage induced from one circuit to another as:
- $v = j\omega M i$.



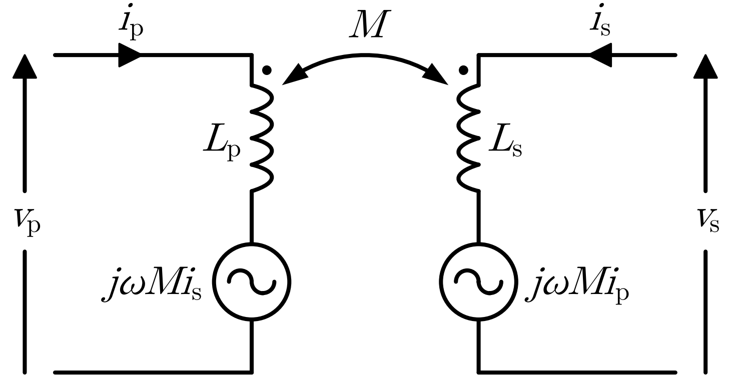
- Ideal transformer in circuit form



- Ideal transformer in circuit broken down into inductance and induced voltages

Magnetics – ideal transformer

- Using the derivations of Faraday's Law from before, an ideal transformer can be described using the following equations:
- $v_p = j\omega L_p i_p + j\omega M i_s$
- $v_s = j\omega M i_p + j\omega L_s i_s$
- L_p is the self-inductance of the primary coil.
- L_s is the self-inductance of the secondary coil.
- This can be re-written in the matrix form as:
- $\begin{bmatrix} v_p \\ v_s \end{bmatrix} = \begin{bmatrix} j\omega L_p & j\omega M \\ j\omega M & j\omega L_s \end{bmatrix} \begin{bmatrix} i_p \\ i_s \end{bmatrix} = j\omega \begin{bmatrix} L_p & M \\ M & L_s \end{bmatrix} \begin{bmatrix} i_p \\ i_s \end{bmatrix}$



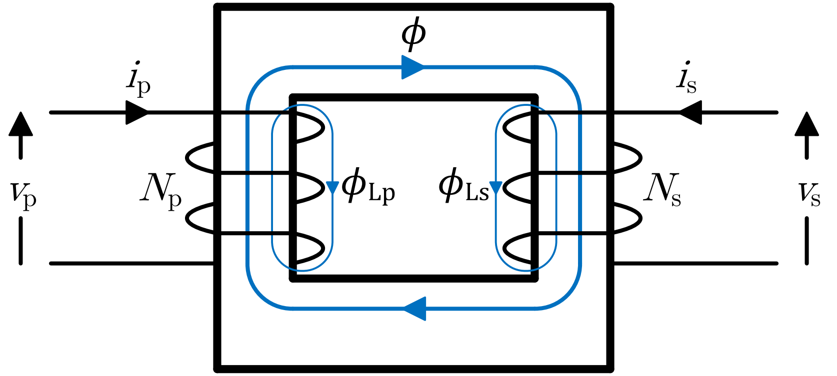
- Ideal transformer in circuit broken down into inductance and induced voltages

Magnetics

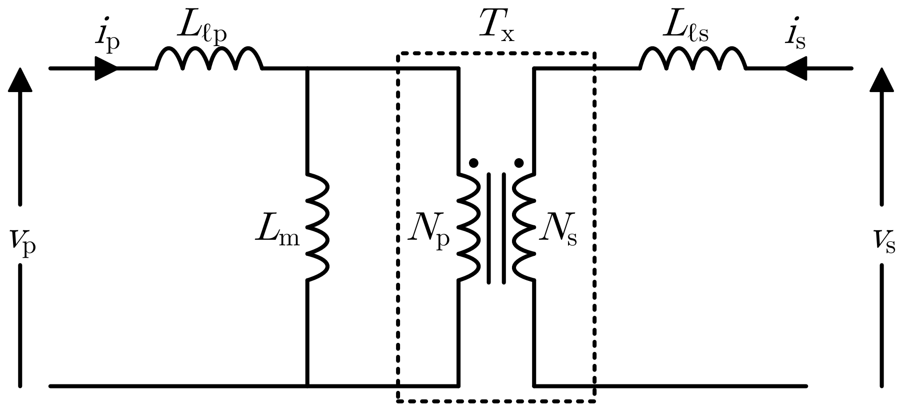
Practical transformer

Magnetics – practical transformer

- In real life, a practical transformer has
  - Losses in the coils and the core
  - Finite relative permeability (μ_r) in the core
  - Leakage magnetic flux (ϕ_{L_p} and ϕ_{L_s}) $\neq 0$
  - Coupling factor (k) $\neq 1$.
- Infinite relative permeability does not exist in real life so some of the magnetic flux generated by the energised coil 'leaks out'.
- This leads the magnetic flux to be divided into two parts:
- **Magnetising magnetic flux** (ϕ_m) passes through the core and is subject to saturation and core losses.
- **Leakage magnetic flux** (ϕ_{L_p} and ϕ_{L_s}) is generated by the energised coil, but does not reach the other coil.



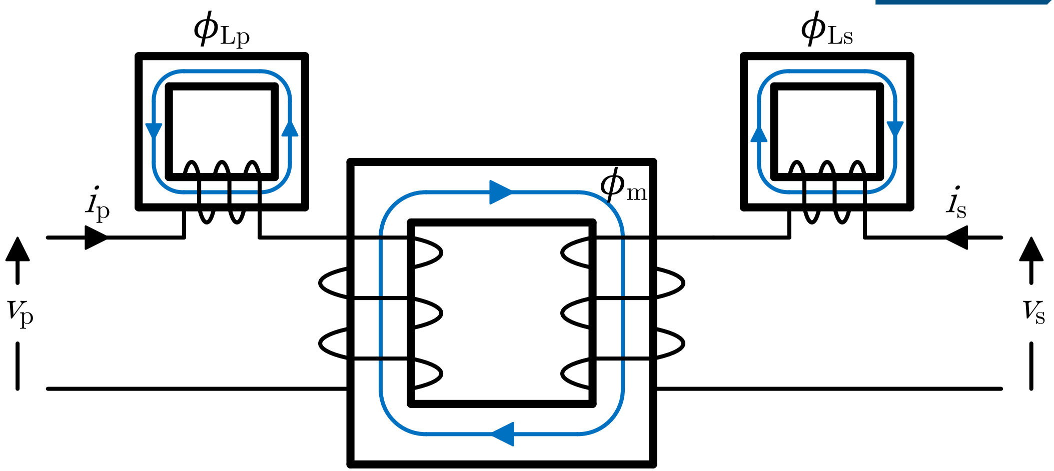
- An example transformer



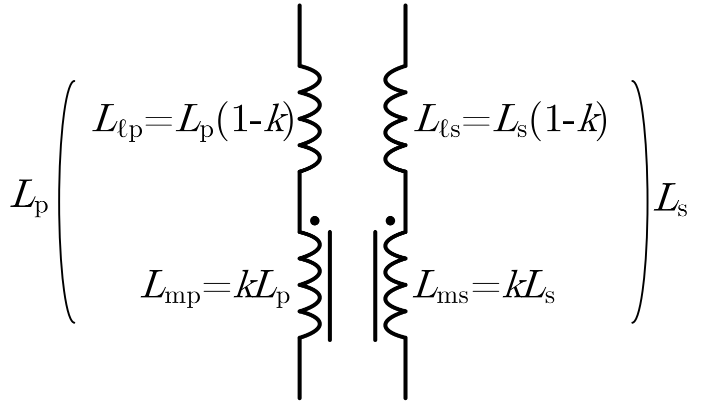
- Ideal transformer in circuit form with practical components

Magnetics – practical transformer

- If the magnetising inductance and leakage inductance could be separated out physically, they would be separate inductors wound in a magnetic core and air cores.
- **Magnetising inductance** (L_{m}) is due to magnetising magnetic flux (ϕ_{m}).
- **Leakage inductances** ($L_{\ell\mathrm{p}}$ and $L_{\ell s}$) is due to the leakage magnetic flux ($\phi_{\mathrm{L_p}}$ and $\phi_{\mathrm{L_s}}$).
- The leakage inductance adds to the total inductance, but does not help with transferring energy.
- In a practical transformer where $(k) \neq 1$,
- $L_{\mathrm{mp}} = kL_{\mathrm{p}}$ and $L_{\mathrm{ms}} = kL_s$
- $L_{\ell\mathrm{p}} = (1-k)L_{\mathrm{p}}$ and $L_{\ell s} = (1-k)L_{\mathrm{s}}$
- For a flyback converter, leakage inductances create issues such as ringing in the switches so should be minimised*.



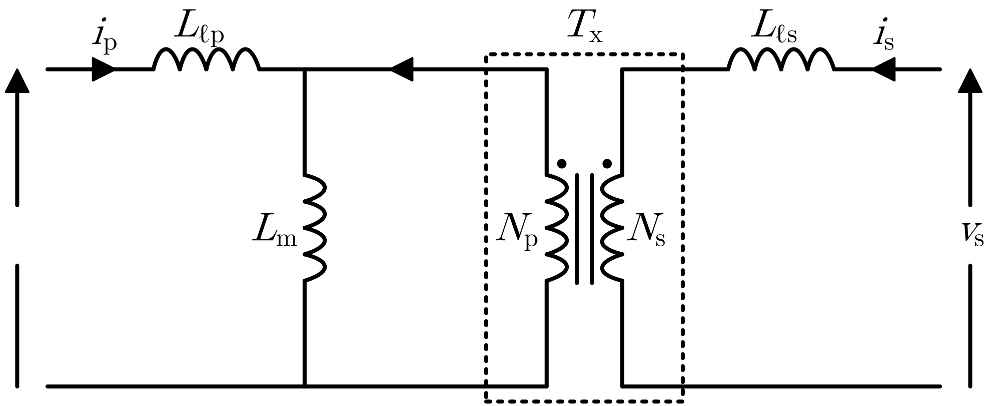
- Representation of separate magnetizing inductance and leakage inductances



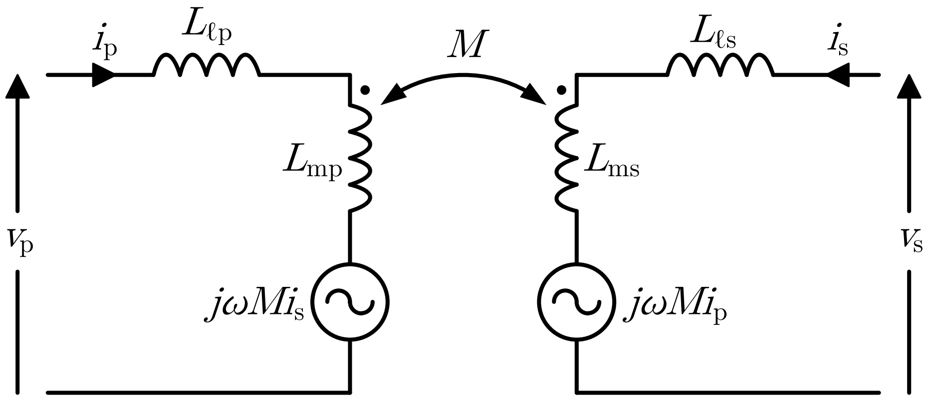
- Division of leakage and magnetising inductances in terms of coupling factor

Magnetics – practical transformer

- The previous matrix with ideal transformer:
- $\begin{bmatrix} v_\mathrm{p} \\ v_\mathrm{s} \end{bmatrix} = j\omega \begin{bmatrix} L_\mathrm{p} & M \\ M & L_\mathrm{s} \end{bmatrix} \begin{bmatrix} i_\mathrm{p} \\ i_\mathrm{s} \end{bmatrix}$
- Substituting: $L_\mathrm{p} = j\omega L_{\ell\mathrm{p}} + j\omega L_\mathrm{mp}$ and $L_\mathrm{s} = j\omega L_{\ell\mathrm{s}} + j\omega L_\mathrm{ms}$,
- $\begin{bmatrix} v_\mathrm{p} \\ v_\mathrm{s} \end{bmatrix} = j\omega \begin{bmatrix} L_{\ell\mathrm{p}} + L_\mathrm{mp} & M \\ M & L_{\ell\mathrm{s}} + L_\mathrm{ms} \end{bmatrix} \begin{bmatrix} i_\mathrm{p} \\ i_\mathrm{s} \end{bmatrix}$
- Mutual inductance and magnetising inductance from the perspective of the primary are related as:
- $M = \frac{N_\mathrm{s}}{N_\mathrm{p}} L_\mathrm{mp}$
- Note that from the perspective of the secondary:
- $M = \frac{N_\mathrm{p}}{N_\mathrm{s}} \left(\frac{N_\mathrm{s}}{N_\mathrm{p}}\right)^2 L_\mathrm{mp} = \frac{N_\mathrm{s}}{N_\mathrm{p}} L_\mathrm{ms}$
- So the matrix can be updated:
- $\begin{bmatrix} v_\mathrm{p} \\ v_\mathrm{s} \end{bmatrix} = j\omega \begin{bmatrix} L_{\ell\mathrm{p}} + L_\mathrm{mp} & \frac{N_\mathrm{s}}{N_p} L_\mathrm{mp} \\ \frac{N_\mathrm{s}}{N_p} L_\mathrm{mp} & L_{\ell\mathrm{s}} + L_\mathrm{ms} \end{bmatrix} \begin{bmatrix} i_\mathrm{p} \\ i_\mathrm{s} \end{bmatrix}$



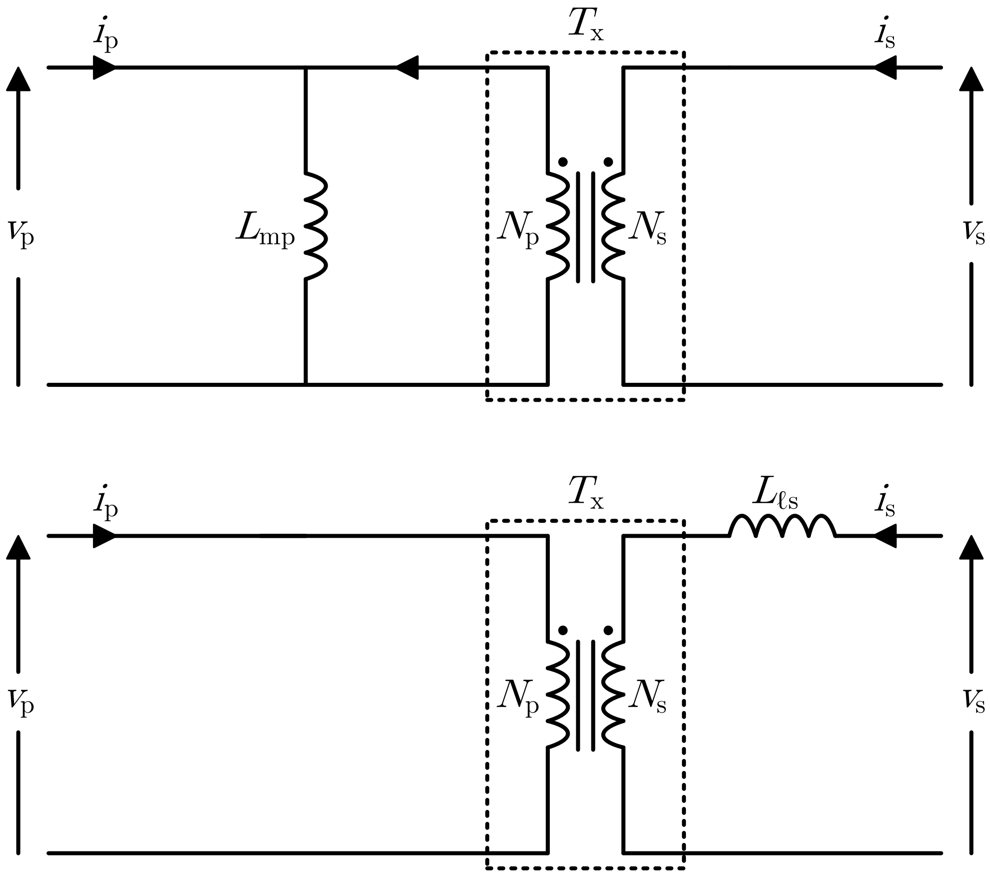
- Ideal transformer in circuit form with practical components



- Leakage and magnetizing inductances broken up

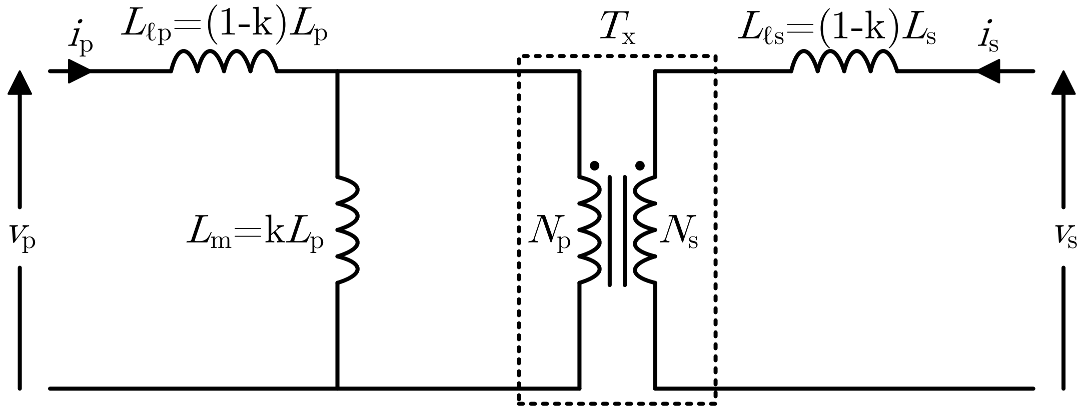
Magnetics – impedance transformation

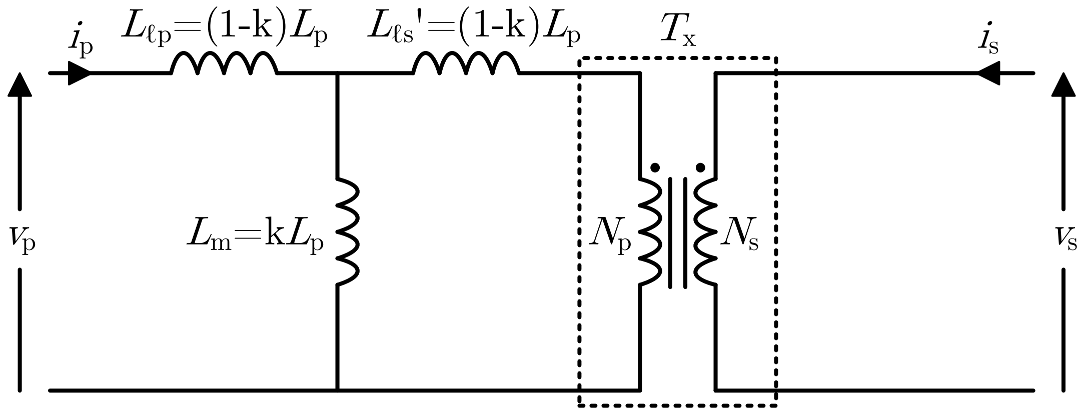
- The impedance for L_{mp} for the primary is:
- $Z = j\omega L_{mp}$
- The ratio for voltages and currents in a transformer:
- $v_s = v_p \frac{N_s}{N_p}$ and $i_s = i_p \frac{N_p}{N_s}$
- The reflected impedance for L_{mp} for the secondary is:
- $\frac{v_s}{I_s} = Z' = v_p \frac{N_s}{N_p} \times \frac{1}{i_p} \frac{N_s}{N_p} = \left(\frac{N_s}{N_p}\right)^2 L_{mp}$
- For the case of $L_{\ell s}$ in the secondary,
- $\frac{v_s}{I_s} = Z = j\omega L_{\ell s}$
- $v_p = v_s \frac{N_p}{N_s}$ and $i_p = i_s \frac{N_s}{N_p}$
- $\frac{v_p}{i_p} = Z = v_s \frac{N_p}{N_s} \times \frac{1}{i_s} \frac{N_p}{N_s} = \left(\frac{N_p}{N_s}\right)^2 L_{\ell s}$



Magnetics – T-model

- To simplify the measurement of the transformer in real life, the leakage inductance in the secondary can be reflected on to the primary.
- The impedance transformation from secondary to primary is:
- $L'_{\ell s} = \left(\frac{N_p}{N_s}\right)^2 L_{\ell s} = \left(\frac{N_p}{N_s}\right)^2 (1-k)L_s$
- Given that $\sqrt{\frac{L_p}{L_s}} = \frac{N_p}{N_s}$, $L_s = L_p \left(\frac{N_s}{N_p}\right)^2$
- $L'_{\ell s} = \left(\frac{N_p}{N_s}\right)^2 (1-k)L_s = \left(\frac{N_p}{N_s}\right)^2 (1-k)L_p \left(\frac{N_s}{N_p}\right)^2$
- $\therefore L'_{\ell s} = (1-k)L_p$
- This is called the ‘T-model’ of a transformer.



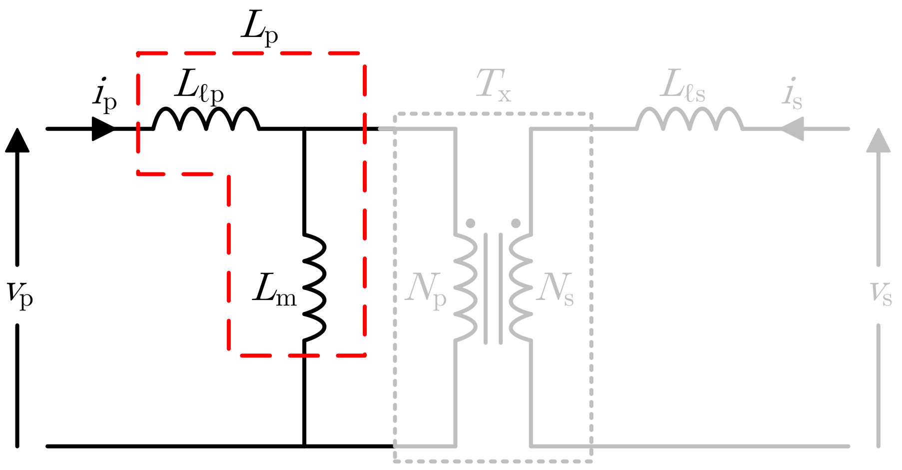


Magnetics

Measuring the transformer

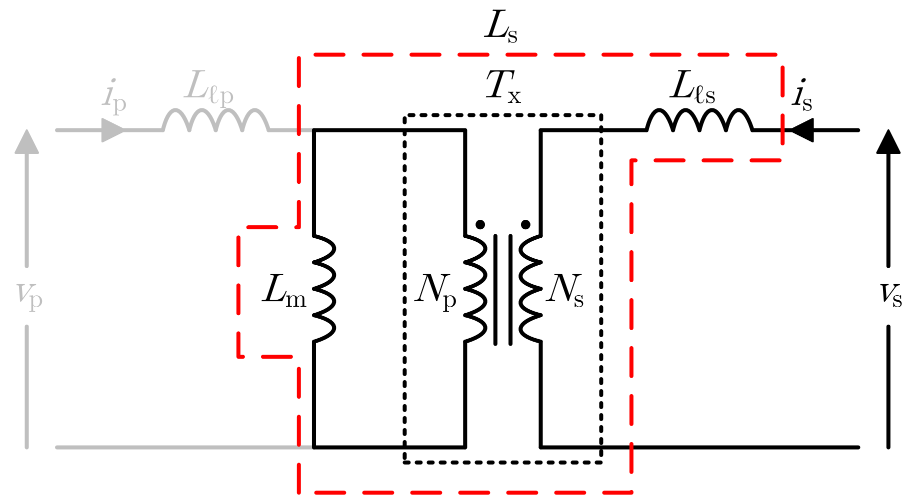
Magnetics – measuring the transformer

- This section looks at deriving the transformer equations without reflecting the secondary leakage into the primary.
- Open-circuiting and short-circuiting the transformer can be used to measure each component.
- If secondary is open-circuited ($i_s = 0$):
- $\begin{bmatrix} v_p \\ v_s \end{bmatrix} = j\omega \begin{bmatrix} L_{\ell p} + L_{mp} & \frac{N_s}{N_p} L_{mp} \\ \frac{N_s}{N_p} L_{mp} & L_{\ell s} + L_{ms} \end{bmatrix} \begin{bmatrix} i_p \\ 0 \end{bmatrix}$
- $v_p = j\omega L_{\ell p} i_p + j\omega L_{mp} i_p + j\omega \frac{N_s}{N_p} L_{mp} \times 0$
- $\therefore v_p = j\omega L_{\ell p} i_p + j\omega L_{mp} i_p$
- $L_p = L_{\ell p} + L_{mp}$



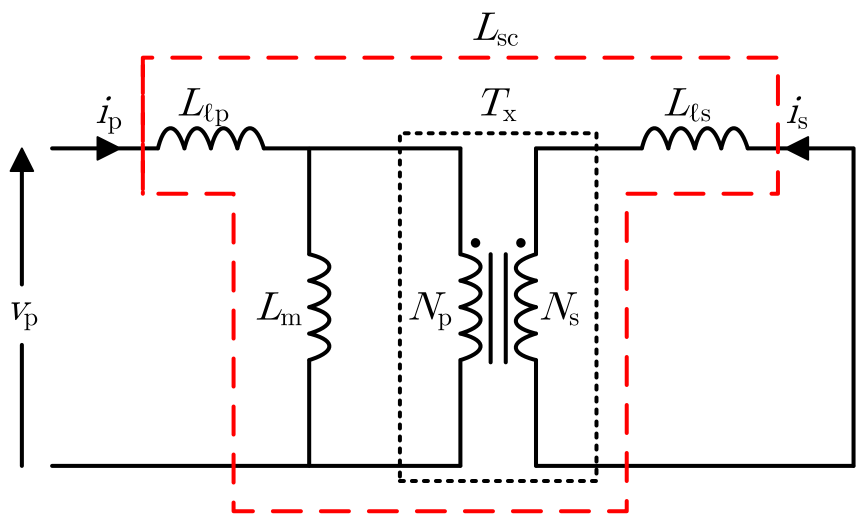
Magnetics – measuring the transformer

- If secondary is open-circuited ($i_p = 0$):
- $\begin{bmatrix} v_p \\ v_s \end{bmatrix} = j\omega \begin{bmatrix} L_{\ell p} + L_{mp} & \frac{N_s}{N_p} L_{mp} \\ \frac{N_s}{N_p} L_{mp} & L_{\ell s} + L_{ms} \end{bmatrix} \begin{bmatrix} 0 \\ i_s \end{bmatrix}$
- $v_s = j\omega \frac{N_p}{N_s} L_{mp} \times 0 + j\omega L_{\ell s} i_s + j\omega \frac{N_s}{N_p} L_{ms} i_s$
- $\therefore v_s = j\omega L_{\ell s} i_s + j\omega \frac{N_s}{N_p} L_{ms} i_s$
- $L_s = L_{\ell s} + \frac{N_s}{N_p} L_{ms}$



Magnetics – measuring the transformer

- If secondary is short-circuited ($v_s = 0$):
- $\begin{bmatrix} v_p \\ 0 \end{bmatrix} = j\omega \begin{bmatrix} L_{\ell p} + L_{mp} & \frac{N_s}{N_p} L_{mp} \\ \frac{N_s}{N_p} L_{mp} & L_{\ell s} + L_{ms} \end{bmatrix} \begin{bmatrix} i_p \\ i_s \end{bmatrix}$
- $v_p = j\omega L_{sc} i_p = j\omega L_{\ell p} i_p + j\omega L_{mp} i_p + j\omega \frac{N_s}{N_p} L_{mp} i_s$
- $0 = j\omega \frac{N_s}{N_p} L_{mp} i_p + j\omega L_{\ell s} i_s + j\omega L_{ms} i_s$
- Rearranging the equation for i_s:
- $-j\omega \frac{N_s}{N_p} L_{mp} i_p = j\omega L_{\ell s} i_s + j\omega L_{ms} i_s$
- $\therefore\ i_s = -\frac{j\omega \frac{N_s}{N_p} L_{mp}}{j\omega L_{\ell s} i_s + j\omega L_{ms}} i_p$
- Substituting i_s into equation for v_p:
- $v_p = j\omega (L_{\ell p} + L_{mp}) i_p - \frac{\left(j\omega \frac{N_s}{N_p} L_{mp}\right)^2}{j\omega L_{\ell s} i_s + j\omega L_{ms}} i_p$

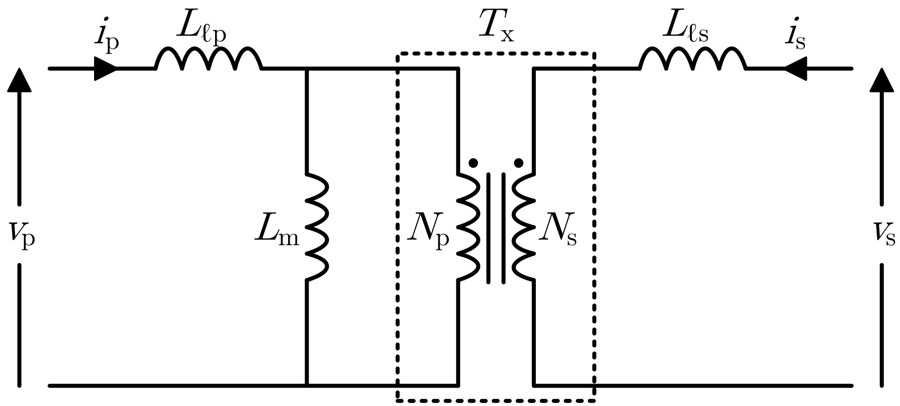


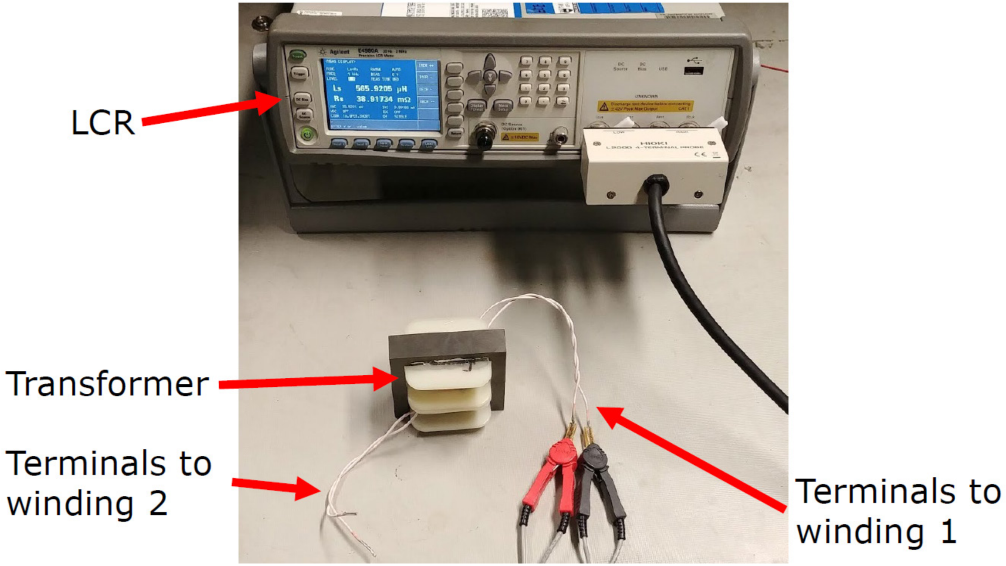
Magnetics – measuring the transformer

- Given that $L_{\text{p}} = L_{\ell\text{p}} + L_{\text{mp}},\ L_{\text{s}} = L_{\ell\text{s}} + L_{\text{ms}}$ and $\text{M} = \frac{N_{\text{s}}}{N_{\text{p}}} L_{\text{mp}}$:
- $v_{\text{p}} = j\omega\left(L_{\ell\text{p}} + L_{\text{mp}}\right)i_{\text{p}} - \frac{\left(j\omega\frac{N_s}{N_p}L_{\text{mp}}\right)^2}{j\omega L_{\ell\text{s}} i_{\text{s}} + j\omega L_{\text{ms}}} i_{\text{p}}$
- $v_{\text{p}} = j\omega L_{\text{p}} i_{\text{p}} - \frac{j\omega M^2}{j\omega L_{\text{s}}} i_{\text{p}}$
- Since $v_{\text{p}} = j\omega L_{\text{sc}} i_{\text{p}}$,
- $j\omega L_{\text{sc}} i_{\text{p}} = j\omega L_{\text{p}} i_{\text{p}} - \frac{j\omega M^2}{j\omega L_{\text{s}}} i_{\text{p}}$
- $L_{\text{sc}} = L_{\text{p}} - \frac{M^2}{L_{\text{s}}}$
- Rearranging:
- $M^2 = \left(L_{\text{sc}} - L_{\text{p}}\right)L_{\text{s}}$
- $\therefore M = \sqrt{\left(L_{\text{sc}} - L_{\text{p}}\right)L_{\text{s}}}$
- Since $M = k\sqrt{L_{\text{p}} L_{\text{s}}}$:
- $k = \frac{M}{\sqrt{L_{\text{p}} L_{\text{s}}}}$

Magnetics – measuring the transformer

- An LCR metre is often used to measure inductances.
- The following can be measured:
- $L_{\mathrm{p}} = L_{\ell \mathrm{p}} + L_{\mathrm{mp}}$
- $L_{\mathrm{s}} = L_{\ell \mathrm{s}} + L_{\mathrm{ms}}$
- $L_{\mathrm{sc}} = L_{\mathrm{p}} - \frac{M^2}{L_{\mathrm{s}}}$
- Given L_{p}, L_{s} and L_{sc}:
- $M = \sqrt{(L_{\mathrm{sc}} - L_{\mathrm{p}})L_{\mathrm{s}}}$
- $k = \frac{M}{\sqrt{L_{\mathrm{p}} L_{\mathrm{s}}}}$





Magnetics – measurement

- Using an LCR metre, the inductance of the transformer can be measured.
- If the secondary is open-circuited, primary terminals measure the self-inductance of the primary winding:
- $L_{oc} = (1-k)L_p + kL_p = L_p$
- Using the turns ratio, the self-inductance of the secondary winding can be determined:
- $L_s = \left(\frac{N_p}{N_s}\right)^2 L_p$
- If the secondary is short-circuited, secondary terminals measures:
- $L_{sc} = (1-k)L_p + \frac{(1-k)L_p kL_p}{(1-k)L_p + kL_p} = (1-k)L_p + \frac{(1-k)L_p kL_p}{L_p}$
- $= (1-k)L_p + (1-k)kL_p = L_p - kL_p + kL_p - k^2 L_p$
- $= L_p - k^2 L_p$
- Then the coupling factor of the transformer can be found by:
- $\sqrt{\frac{L_{oc} - L_{sc}}{L_{oc}}} = \sqrt{\frac{L_p - L_p + k^2 L_p}{L_p}} = k$
- From here, $M = k\sqrt{L_p L_s}$